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A Fresh Look at the Principles of Motion Economy

by **RAYMOND N. BLAIR**

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Carnegie Institute of Technology

THE various collections of miscellaneous rules and precepts commonly referred to as "the principles of motion economy" are more or less direct descendents of a work published by the Gilbreths in 1923 (2). Its modification and development by Ralph Barnes (1) into an excellently illustrated set of twenty-two classified principles seems to be one of the formulations most favored by subsequent Industrial Engineering writers, including those who refer to this area incidentally—e.g., Robert Roy in printing management (4) and J. Stanley Gray in industrial psychology (3).

However, as various commentators have noted (4), (5) and as Barnes himself recognized and stated (1), these various listings do not represent the principles of motion economy in any fundamental sense, but simply provide some useful rules, of varying significance and of unequal importance. These range from basic points of general applicability, such as the preferability of smoothly continuous motions over motions involving sudden and sharp changes in direction, to very specific suggestions of the type describing a good kind of handle for large screwdrivers. Attempts to classify this miscellany have been only partially useful. For example, the rule stating that when individual fingers perform specific movements, the load should be distributed in accordance with the inherent capacities of the fingers, which is usually listed under "Motion Economy as Related to Design of Tools," could with equal reasonableness be classified under another heading, "Motion Economy as Related to Use of Human Body." Some of the rules listed are actually practical examples of several underlying principles; some examples used to illustrate other points bring out principles more fundamental than those expressed in the primary listing.

One of the sources of difficulty is the conflict between generality at the expense of practical usefulness and specific "handy hints" at the expense of universality of application. This ambiguity of purpose accounts for the different points covered in different listings and the heterogeneous nature of the "principles" specified in any particular formulation.

A considerable study of the subject has resulted in the development, here presented, of a systematized approach having three stages or levels. All the fundamental characteristics of economical motions and their efficient coordination are presented in the first part. Since the objective at this point is clear, approximate unanimity regarding the items included here can probably be attained without much difficulty and with little or no further research. A task is performed with motion economy when, at a given work-pace, the time required is at a minimum and the resulting fatigue is at a minimum. The characteristics of motions with respect to their time-consuming and fatiguing properties are obviously related to their number, length, shape, muscular force requirements, distribution among the members of the body, coordination, etc. The fifteen principles listed below have not all been expressed in so basic a form before; however, most of them have been implicit in the specification of some of the techniques through which they may be achieved.

The second part consists of a presentation of those human physiological characteristics which have a bearing on motion-time and fatigue. Part three discusses the general techniques which have been evolved for realizing the objectives expressed in the fundamental principles, and is the place where specific applications for illustration purposes may be most suitably located in a thoroughgoing exposition. Both of these latter parts are subject to indefinite expansion and development, and represent the areas having greatest potentialities for further investigation and research.

Several advantages proceed from this distillation and separation of the basic principles from techniques, special applications, and other accouterments. Greater clarity and coherence is achieved for teaching purposes. The possibilities and needs for further research are better defined and more forcibly brought to attention. A pattern for exposition, flexible for greater or lesser degrees of detail without omission of important fundamentals, is provided. Finally, it is hoped that greater facility in practical application may result.

FUNDAMENTAL PRINCIPLES OF MOTION ECONOMY

The order of presentation follows a plan of listing first the principles referring to individual or successive motions and last the principles referring to the co-ordination of concurrent motions. Principles 9, 10, and 11 may be in either category, since the work may be performed chiefly by the eyes, as in proofreading, or the eyes may direct or aid in the co-ordination of other members of the body performing the main work.

1. Minimize the number of motions.
2. Minimize the lengths of motions.
3. Provide for the use of continuous-curved motions.

4. Minimize the number of parts of the body involved in a complex motion,[1] or change the complex motion to a simple motion.

5. Minimize the necessity of motion control by muscular effort.

6. Minimize muscular force required for motions.

7. Provide for constant motion paths which encourage the quick establishment of effective habit patterns.

8. Provide for easy and natural motions with respect to the physiological characteristics of the body.

9. Minimize the number of eye fixations required.

10. Minimize the distance apart of eye fixations required.

11. Minimize the eye fixation time required for perception when this is a controlling factor.

12. Distribute actions among the members of the body in accordance with the inherent capacities of the members.

13. Provide for intermittent use of different muscles.

14. Provide for simultaneous motions of both arms.

15. Provide for symmetrical motions of both arms, or (next best) right-angular motions, or (less satisfactory) unidirectional motions, or (least satisfactory) dissimilar motions.

Reduced thus to uniform fundamentals of general applicability, this list, with possible minor modifications, seems complete and in essence almost invariable. There may, of course, be differences in form. The grammatical mood might well be different. There could be combination, condensation. The whole list, in fact, might conceivably be expressed in a single sentence ("Economical motions are few, short, curved, minimal in body movement, uncontrolled . . ." etc.). Separation into elements each consisting of a single numbered factor has been done for completeness of expression and convenience of reference. But the elements themselves could hardly be much different.

The supplementary material, on the other hand, seems susceptible of almost unlimited development, improvement, and addition. The following fragments are intended merely to illustrate the nature and organization of the material in these areas.

PHYSIOLOGICAL CHARACTERISTICS AFFECTING MOTION ECONOMY

Fingers. The strongest and most accurate motions of the fingers are those directed inward (toward the palm).

Relative work-load capacities of the fingers of the average individual (involving both the factors of strength and endurance) are approximately as follows: Right index and middle finger capacities are greatest; right ring and little and left index and middle are about 10% less; while left ring and little finger capacities are about 20% less than right index and middle finger capacities. (Figures of much greater precision are available for such purposes as keyboard design.)

Arms and hands. The easiest, most natural movements of the arms and hands with respect to a horizontal plane at elbow height are: 1. for motions toward and away from the body, along a path (slightly curved) making approximately a 60° angle with a line parallel with the shoulders; 2. for motions rotating about the elbow, along that part of the arc nearest the body, with rapidly diminishing ease along the outer part of the arc away from the body. (The well-known diagrams of normal and maximum working areas are applicable at this point.)

When the body is in a normal standing position, the most forceful movement of the arm is upward, bending at the elbow, from a fully-extended downward position.

Free or ballistic movements are engendered by the action of only a positive muscle group, while controlled motions involve an additional set of opposing muscles, resulting in greater fatigue.

Eyes. The eyes move in quick jumps followed by fixations of sufficient duration to permit perception. Eye travel time depends upon distances involved and sureness about location. Eye fixation time is a function of several factors, chiefly size, brightness, color contrast, adequacy of illumination for the particular situation, and in some cases intellectual processes that may be involved in perception. Black on yellow provides the most distinctive contrast, and yellow is the color most readily identifiable under different lighting conditions.

TECHNIQUES FOR ACHIEVING MOTION ECONOMY

Methods Design. A balanced two-handed operation, which can often be achieved most effectively by having two operation cycles performed simultaneously, one with each hand, helps realize the objectives specified in Principles 14 and 15.

Using momentum developed in one action to assist the performance of a following action is one way of applying Principles 5 and 6.

Methods designed to employ the force of gravity to do some of the work may help achieve Principles 1, 2, 6, and possibly others.

Layout Arrangement. "Circular" work-place layout, with close-in, fixed locations and positions for tools and materials, arranged in reference to frequency of use and sequence of operations, provides excellent opportunities for making use of Principles 1, 2, 4, 7, and 9. (Further use of the working-area diagrams is appropriate here.)

Equipment Design. Simple fixtures or holding devices often assist the design of a balanced two-handed operation and also help achieve the objective expressed in Principle 1 by eliminating reactive motions. If one hand is used as a holding device for an object upon which the other hand works, the action of the first hand is an example of a *reactive* effort: it is opposing the force of gravity and also the action of the other hand, and is

[1] A complex motion is here defined as one in which the various parts of the body involved are moving by muscular control in different directions or about different centers of rotation. For example, a fifth-class reach is necessarily complex, for the forearm rotates about the elbow, the upper arm about the shoulder, and the upper body perhaps at the waist. A third-class motion, on the other hand, may be simple or complex. A telegraph key being tapped by use of the finger, hand, and forearm acting as a single unit, albeit loosely, is a simple motion and a good one. Writing by means of finger motions alone (first-class motions) involves, on the other hand, complex motions of the fingers.

therefore performing at least as much fatiguing work as if it were productively engaged.

Guides and stops help realize the objectives of Principles 5 and (sometimes) 9. Guides minimize use of eyes and muscles for *direction* control, and stops eliminate the need for *deceleration* by muscular control, as the kinetic energy is dissipated abruptly against the stop. Both devices thus permit faster accurately-directed motion with less fatigue.

Gravity feed bins and drop deliveries are two important physical aids to the employment of gravity in methods design.

Foot-pedal or knee-actuated devices are often capable of relieving the hands of certain operations, in accordance with Principle 12.

Tools and machine controls should be designed and located for most effective use. Combined tools often reduce the number of motions (Principle 1) by eliminating some lay-downs and pick-ups. The handles of screwdrivers used in heavy work should be of large diameter, for this increases the "mechanical advantage" and thus reduces the force required for turning (Principle 6). Machine controls should be located so that their use permits taking best advantage of the physiological characteristics of the body as discussed. Mirrors may sometimes be used to facilitate observation of the results of adjustments and for other purposes.

Working Conditions. For all operations in which the eyes do the major work or direct and co-ordinate other motions, adequacy of illumination has a vital influence on the time required and fatigue produced (Principle 11).

When the height of the work-place and chair can be arranged to provide for alternate sitting and standing, different sets of muscles are alternately used and relaxed (Principle 13).

CONCLUSION

Although there is doubtless room for some disagreement about the completeness and/or accuracy of the fifteen fundamental principles presented here, it is hoped that this discussion will lead at length to a widely-accepted standardized formulation and that renewed interest and development will be stimulated in an area which, despite the rapid advancement of automation, will continue for a long time to be a field of major importance to Industrial Engineers.

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Management: A Goal for Industrial Engineers?

by **H. G. THUESEN**

Professor, School of Industrial Engineering and Management, Oklahoma State University

MUCH has been written about the need for and the shortage of engineers. Emphasis has been upon the shortage of personnel competent in the physical technologies. Some have held that engineers should keep to research and design and leave management to others. On occasion it has been implied that promotion of engineers into supervisory or managerial positions is not in the best interest of society. Acceptance of executive positions by engineers has in some quarters been regarded as a breach of professional loyalty. The writer remembers a story in this *Journal* on what to do about AIIE members who had forsaken technologies of Industrial Engineering for leadership positions.

The fact that salary schedules for engineers engaged in leadership capacities are generally better than schedules for those engaged in research and design suggests that shortage of leadership talent is more urgent than research or design talent. This viewpoint seems to be growing.

In a recent competently made survey by The Professional Engineers' Conference Board for Industry in cooperation with the National Society of Professional Engineers the engineers surveyed were asked: *"Assuming an engineer could advance just as far in strictly engineering work as he could by moving into management, which way would you personally prefer to direct your career?"* The response was:

Strictly engineering	34 per cent
Management	50 per cent
Qualified answers	9 per cent
No opinion	3 per cent

To the question, *"If your company sponsored training in all of the things on this list, which three do you feel would be most profitable to you?"* In order of preference the response reported was:

Organization & Planning	66 per cent
How to Handle People	50 per cent
How to Supervise	41 per cent
Public Speaking	38 per cent
Letter & Report Writing	35 per cent
Company Economics	34 per cent
Advanced Engineering	31 per cent
How to Conduct Meetings	25 per cent
Advanced Mathematics	18 per cent
Budgeting	14 per cent
None, no Choice	5 per cent

A single swallow does not make a summer nor do two items in a survey tell everything about engineers' aspirations. They do indicate that a significant percentage, if not a majority, of the engineering-trained feel that they would prefer a career broader than research, design and related activities.

It may be noted that most of the areas of study given high preference in the table above are areas common to Industrial Engineering training and experience. Could the preference for managerial careers be the reason that the number in Industrial Engineering has increased more rapidly in the post war period than that of any other engineering group? Could it be the reason behind the phenomenal growth of AIIE? Be that as it may, it is certain that Industrial Engineering is receiving increasing recognition in the industrial world.

This brings us to the question in the title, *Management:* A Goal for Industrial Engineers? Well why not?

Particularly since the Industrial Revolution man has realized his limitations and sought to circumvent them by employment of machines. Engineering branches have been built about these devices and the phenomena that underlie them as for example, chemical, electrical, mechanical, automotive and aeronautical engineering. These truly marvelous developments are basically labor saving. They permit greater accomplishment per unit of applied human muscle and mental effort.

But as marvelous as are physical devices, man's greatest invention is organization. This is the means whereby man joins his efforts with that of his fellows to do what cannot be done at all or cannot be done economically by individual action. In illustration of the labor saving characteristics of organized action consider the following illustration: The owners of two adjacent cars find that snow prevents them from getting their cars in motion. Each realizes that he can attach chains to the car wheels and estimates that this will require 30 minutes. Each also realizes that if he helps the other by pushing, the car can be gotten under way in an estimated time of one minute each. Thus, by working as an organization $(30 \times 2) - (2 \times 2)$ or 56 man minutes of labor may be saved.

Whereas, the emphasis of other branches of engineering is upon machines, Industrial Engineering primarily concerns itself with the fruitful functioning of human effort in organizations. The activity whereby human effort in organization is made effective is designated by such terms as leading, supervising and managing. Managing is a creative activity in a dynamic situation. Its creativity is illustrated by the following example.

Suppose that 100 men had been stumbling over 100 stones in a path they used. Suppose each man in turn had considered the matter of removing the stones. Each had found that it would be worth $5 per stone to him to be free of the stones and that it would cost $50 per stone to have them removed. Each path user abandoned the stone removal project as uneconomical and the 100 men continued stumbling over the 100 stones. A manager then came upon the scene. He figures the cost of removing 100 stones at $50 per stone to be $5000. He calculates the potential benefit to be $5 for each of 100 stones for each of 100 persons or $5 $\times$ 100 stones $\times$ 100 persons or $50,000. Thus, there is $45,000 to distribute between the 101 persons involved in the "organization" as a result of the creativity of a manager.

The function of the managerial activity is to direct the talents and efforts of men in combination to be more fruitful than individual effort. Effectively managed organizations are means for creating and exchanging utilities. This may be illustrated by considering a business organization. Each employee of the organization must value the efforts he contributes less than the wages and other benefits he receives in return or he surely would withdraw. A vendor must value the materials or equipment he sells to the organization less than he values what he receives in return for them or he would not sell. The financer values the funds he advances now less than the funds he receives in return later or he surely would not continue to finance the organization. Finally, the customer must value his money less than the products of the organization he buys or he would not buy them.

To be successful the total of satisfactions an organization distributes must be greater than the total contributions it receives. Organizations are essentially devices to which people contribute what they value less to gain what they value more. Unless persons, in terms of their evaluations, get out more than they put into an organization they withdraw from it. For an organization to endure, its efficiency measured in satisfactions (output divided by input) must exceed unity.

The function of managerial action is to maintain a system for the pooling of the activities of people so that each person gains more than he contributes, and such that he does not believe a superior alternative is open to him.

A manager plays his role by:

1. Seeking out objectives which can be reached by the pooling of human effort and which will result in the creation of utilities in excess of the burdens they impose on contributors.

2. Determining the specialization of human acts with regard to time, place, method, etc., needed to attain the objective.

3. Motivating persons to perform the needed acts by distributing utilities that are created to them.

Persons who engage in leadership activities are the key to the creativeness of group action. Excellence of management is a widely recognized primary competitive factor in modern free enterprise. Most competitive factors other than management such as equipment, personnel, markets and raw materials are substantially equally available to industrial organizations. Management or leadership is the strategic factor in all group action. As measured by rates of compensation leadership ability is more in demand than strictly technical ability.

This is an age of group action. For example, education, research and production are carried on primarily in organizations. And it is managerial action which directs the application of mathematics, science and operating techniques and practices in the accomplishment of organizational objectives.

Organizations are no less complex nor dynamic than machines. For many, managerial activities will be more fascinating and challenging than effort related primarily to the physical sciences.

By training and experience the Industrial Engineer is in an excellent position to advance in leadership activities. The application of Industrial Engineering techniques acquaints him with centers of management control. He is in a favorable position to use Industrial Engineering techniques and experience as stepping stones to progress into management.

Management may well be a goal for Industrial Engineers because of the challenge, prestige, compensation, broad outlook and opportunity for accomplishment this dynamic activity provides.

Walter Rautenstrauch's Pioneering Work— A 50th Anniversary

by **RAYMOND VILLERS**

Rautenstrauch and Villers, Consultants to the Manufacturing Industries

The following article has been prepared at the request of the Editorial Staff. We feel very fortunate that Dr. Raymond Villers, long time friend and associate of Walter Rautenstrauch, could be persuaded to prepare the material.—Ed.

THE present year is the 50th anniversary of one of the most significant contributions made to the growth and development of Industrial Engineering. This contribution was the step taken by Walter Rautenstrauch when he offered his course on *Works Management* at Columbia University during the academic year 1907-08.

Walter Rautenstrauch, until his sudden death in 1951, at the age of 70, was one of the most active and reliable experts in Industrial Engineering. Even after he had passed retirement age and had become Professor Emeritus, he continued to teach and, in cooperation with the author, wrote more books and continued his consulting work and his research work. At the time the AIIE was founded, he demonstrated his dynamic good judgment by being one of the first to join.

Walter Rautenstrauch is not only well known for such a successful career. He is also recognized as having been a pioneer in the field of Industrial Engineering. He was the founder of the Department of Industrial Engineering at Columbia University and the head of the department for some thirty years. It was at Columbia, where he was at the time a Professor of Mechanical Engineering, that he started in 1907 his course on *Works Management*.

Walter Rautenstrauch's course was offering what can be called the *broad concept of Industrial Engineering*. This was then an entirely new concept. Much had already been accomplished by the Scientific Management Movement. Great progress had been made in the direction of improving the efficiency of production methods, of plant organization and that of various special activities within the industrial enterprise. No attempt had been made, however, to integrate effectively each and every activity related to industrial production. Such an effective integration is precisely the goal of the broad concept of Industrial Engineering. Today, this concept is best expressed by the AIIE definition of Industrial Engineering adopted in 1956 by the Seventh National Convention:

> Industrial Engineering is concerned with the design, improvement, and installation of integrated systems of men, materials, and equipment. It draws upon specialized knowledge and skill in the mathematical, physical, and social sciences together with the principles and methods of engineering analysis and design to specify, predict, and evaluate the results to be obtained from such systems.

The adoption of this definition has been very stimulating to the Industrial Engineering profession. It has provided the solid framework of reference needed by everyone who works in the field. It has also greatly helped to

clarify a controversy that originated with the start of "Operations Research." This controversy can now be called a quarrel in semantics. As of today—and this is probably due to a large extent to the adoption of this definition—Industrial Engineering is recognized as being concerned with all aspects of *industrial operations*. There is also an increasing recognition that Operations Research is actually *a continuation of the best in Industrial Engineering* (1).

Looking back to the origin of this trend toward recognition of the broad concept of Industrial Engineering, it is interesting to note that these recent developments coincide with the 50th anniversary of the pioneering step taken by Walter Rautenstrauch when he offered his new course on *Works Management* that was presenting this broad concept for the first time in history.

The pioneering character of Rautenstrauch's course was promptly recognized. Before the year 1908 was over, *Engineering Magazine*, a leading publication of the time, stated in its editorial:

> Provision has been lacking for the large proportion of engineering graduates whose future work lies in the intelligent, efficient direction of manufacturing operations. It is most gratifying to find that two of the great American Universities, Columbia and Harvard, are undertaking to solve the problem, one from the engineering and the other from the economic side (2).

Columbia's "Engineering side" was Walter Rautenstrauch's course on Works Management. What was new in his approach was the attempt made for the first time to teach as one single subject the fundamentals of what is today recognized as the broad concept of Industrial Engineering. The course's announcement read as follows:

> *ME52—Works Management*
>
> Methods of manufacture and output affecting systems of management of mills and factories and cost of production. Departmental division and distribution of authority and responsibility as affecting the directness of producing. Departments of engineering, construction, sales and stores. Executive department. Methods and forms of reports to directors and various departments. Labor accounting systems. Cost determination and selling price. Purchase of raw material and sales of products. Utilization of scrap and waste to product. Layout of mills and character of construction with regard to lighting, heating, ventilation, sanitation, insurance and reduction of fire hazard. Group and individual drive. Principle of mill construction and power.

It is interesting to compare this announcement to the above quoted AIIE definition of Industrial Engineering. The terminology used is bound to be different, if only because half a century had gone by, but there is a remarkable analogy in the spirit that is behind the words used in each case. The AIIE definition uses the terminology "integrated systems of men, materials and equipment." The course focused attention upon the "systems of management" and followed this terminology by a description that clearly implied integration of *men* ("distribution of authority and responsibility"), *materials* ("Purchase of raw material and sales of products") and *equipment* "Layout of mills and character of construction, cost determination, etc."). The AIIE definition speaks of "social sciences"—the course mentioned the subject of "Group and Individual drive."

It thus appears that this course on *Works Management*, offered in 1907, constituted an exceptional pioneering step in the establishment of Industrial Engineering.

The course did not specifically mention, as the AIIE definition does, the relationship of Industrial Engineering with mathematical sciences. Yet, in the years that followed, Walter Rautenstrauch was also going to become a pioneer in the development of this specific relationship. As early as 1910, he was advocating the use of a mathematical approach very similar to what we would today call Linear Programming. He reported having made use of such a method in an article he published in the *ASME Journal* (Vol. 32, 1910), "A Comparison of Lathe Head-Stock Characteristics."

In this article, Walter Rautenstrauch was considering the problem of inquiring into the relative merits of two lathes for use in rough turning forgings. He noted that the capacity of a lathe for rough turning depends on the speed and the area of cut it can take on all diameters of work it is able to swing. The area of cut in turn will depend on the torque which may be delivered to the spindle. Accordingly, if a torque speed diagram is set up for each lathe, their relative capacities for doing rough turning may be judged by a comparison of these diagrams.

Such an approach as the one thus advocated is typical of the use of mathematical methods for the solution of *specific problems* such as the selection of the most advisable piece of equipment.

A few years later, Walter Rautenstrauch went one step further. His purpose was to accomplish what we would today call "overall profit optimization." To serve this purpose, he offered as a tool of management his now famous *break-even chart*. The break-even chart provides an analysis of the *overall situation* and does it on the basis of a mathematical study of the cost to sales relationship. Actually, Rautenstrauch's break-even chart appears to have been the first presentation of what we would today call a *mathematical model*.

These two aspects of his research work, the break-even chart and the approach offered for the selection of a lathe, illustrate a fundamental characteristic of Rautenstrauch's pioneering work, namely, his consistent effort to associate technological talent with an understanding of financial considerations. His ultimate goal was to provide a technically sound and financially profitable approach to Industrial Engineering

One may well ask the question: "What led Walter Rautenstrauch in such a pioneering direction?" The answer to this question is itself related to another set of very recent developments. Today we hear a lot about the need for a modern education, the need for a balance be-

tween culture in the traditional sense of the word and a technical training related to the most advanced scientific developments. It happened that in this respect Walter Rautenstrauch was quite a pioneer in terms of his own cultural background. This is probably what made him a pioneer in Industrial Engineering. His personality was characterized by the conjunction of his deep faith and interest in human nature and of his highly developed technical skill. He was a mechanical engineer who soon became a recognized expert in machine design. He was also a cultured philosopher deeply concerned with the structure of the human mind and the problems of human relations.

This remarkable and constructive combination is probably what stimulated his interest in the Scientific Management Movement started at the turn of the century by Frederick W. Taylor, Henry L. Gantt, Horace K. Hathaway, Harrington Emerson, the Gilbreths and their associates.

Very soon, Walter Rautenstrauch decided to help develop what he considered as a new branch of engineering and liked to describe it as *the design of the manufacturing enterprise*.[1] He was impressed by what he considered as a fundamental similarity between machine design and the design of the manufacturing enterprise. He stressed the fact that both the machine and the enterprise are based upon an integration of mechanical devices and human skills and that both the machine and the enterprise must be operated economically. It is this fundamental consideration that led Walter Rautenstrauch to believe that the best contribution he could offer to the Scientific Management Movement would be to stress the need for a broader concept of Industrial Engineering. A few years after having started his course on *Works Management*, he had an opportunity to emphasize once more this need for a broad concept. In 1912, the Society for the Promotion of Engineering Education called a meeting for the specific purpose of "showing what scientific management is and what it can do." The meeting was held in Boston under the chairmanship of Frank B. Gilbreth. The speakers included, among others, Harrington Emerson, H. L. Gantt, H. K. Hathaway, Hugo Diemer and Walter Rautenstrauch. It was in fact a meeting of almost all of the pioneers in the field.

At this meeting, Rautenstrauch presented a paper on the subject of *Teaching the Principles of Scientific Management*. In this paper, he stressed again the need for a broad concept of Industrial Engineering. He also indicated in the following terms how this broad concept is related to the analogy he was making between machine design and the design of the manufacturing enterprise:

> It is indeed important to teach students to lay out machines, to determine the forces in the mechanism, and to proportion the parts for stiffness and strength, but it is equally important that they be taught that a variety of machines might be designed to perform precisely the same service, and that there will result perhaps varying degrees of goodness but certainly great variations in cost not only to build them but to operate them. Unless a machine can be built at a cost sufficiently low to meet the demand it will never get beyond the experimental stage and there is absolutely no use in designing a machine if its cost of production or operation exceeds what can be properly paid. It is certainly important to teach the students how a bar can be turned round or a plate made flat or how every operation necessary to making a machine may be carried out by a mechanic; but it is equally important that he should know their relative costs so that he may avoid the expensive operations or reduce their use to a minimum. He should also be taught that the maximum economy in producing that machine will result when it is made in quantity, and why, and that quantity production will involve changes in design, changes in shop processes, because it permits the use of special instead of standard tools and finally and most important, that quantity production involves many men and that many men require management that each may produce the maximum for his wages, and that such an organization for economic production is subject to laws and principles of far greater consequence than all the thermodynamics that was ever formulated; and yet laws of essentially the same nature as those that control the design of his machine (3).

Rautenstrauch's effort to stress in these terms the analogy and also the close relationship between machine design and the design of the manufacturing enterprise tends to take an even greater significance in our days of "automation." In any case, the implications of this analogy and of this close relationship are now fully acknowledged. Today, methodical organization planning is based upon the scientifically designed integration of technical, economic and human factors. Such an integration is the ultimate goal of the broad concept of Industrial Engineering. It is gratifying to see that full recognition is now given to such a concept. This recognition has made Walter Rautenstrauch a true pioneer in the field.

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In Memory

DWIGHT D. GARDNER

First Elected National President

of

American Institute of Industrial Engineers

[1] This became the title of one of his books, published in 1941 which was well read throughout the country.

The Equivalence of Rating Motion Picture Films and Actual Operations

by **RICHARD W. McGUIRE**

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San Diego, California

FREDERIC W. TAYLOR recognized that individuals differed in their work habits and their production capacities. His attempt to give the worker and the company the benefit of abilities by worker selection failed miserably. His technique has since been taken up and expanded by Industrial Psychologists through aptitude tests. Worker selection was almost completely dropped by Industry after the congressional investigation of "scientific management." The abandonment of this technique left a void in the time study system which was not filled until the introduction of leveling. Leveling attempted to categorize a worker along a continuous scale so as to adjust his time to coincide with that of a hypothetical "average operator." This addition to the time study methodology met with immediate approval and adoption. One of the major steps even today is to adjust the observed time so as to coincide with that of a hypothetical "average operator." Rating is the contemporary technique most used to accomplish this feat. It consists of an observer making a judgment as to how the observed worker ranks along a pre-established scale or base on which the 100 (sometimes 60) represents the key or base point.

Gilbreth introduced motion pictures to industry to study the minute and elemental motions comprising a task. Despite the high costs, films offered such advantages that they were rapidly accepted as an industrial tool. They have, following their introduction, been used for 1. motion study, 2. recording of events, 3. training time study men to rate, 4. a base for rating comparison, etc.

STATEMENT OF PROBLEM

The important point is that films have been used directly and indirectly to draw a judgment about a separate action. It is true that the film has captured the motions observed on a two dimensional plane. However, the important question concerns the equality of judgment made from these two mediums. If a time study man observed and rated a job in the shop, would this rating be the same as if he had rated a film of this same operation taken at the time he had rated the actual? We need to know whether or not the judgments (ratings) made from the film are equivalent to those made from the actual operation.

DESIGN OF EXPERIMENT

An experiment was designed to test the hypothesis: "Rating of motion picture film is equivalent to rating actual operations." To test this, results from rating films and rating actual operations must be compared and any differences tested to determine their reality. This was accomplished by designing the experiment around a complex triple classification analysis of variance. A group of twenty advanced students were trained extensively in rating for a period of three months. The training was done on both film and actual operation. The group was divided into two equally matched groups at the end of the training period. This was accomplished by weighing and summing historical rating data for each individual. This result furnished an order of rank between the students so that the selection of two equally matched groups evolved to a mechanical process—the first was placed in Group I, the second was placed in Group II, the third was placed in Group I, etc. It was now possible, with the two matched groups, to have one group rate film while the other rated actual operations. However, if matching was not perfect, an error would be introduced. To guard against this the design was so made that each group could rate both film and actual operations.

Two plants were chosen in which to study jobs. Thirteen jobs in the first plant were selected for rating and filming; in the second plant ten jobs were chosen. These jobs had work cycles of less than one minute, were predominantly man paced and were confined to a limited working area. In fact, all operations were chosen so as to include the above requirements. The jobs were observed and rated in two periods in each plant. Group I observed and rated the first half of the jobs in Plant I while pictures were being taken; Group II rated these films. Group II observed and rated the second half of the jobs in Plant I while pictures were being made; Group I rated these films. The same system was followed in Plant II.

The ratings for all cases were collected and arrayed so that the ratings for films formed one column and the ratings for the actuals formed another column. An analysis of variance was performed which yielded the results shown in Figure 1.

RESULTS

This brings out two very important items. First, differences do exist between the ratings of the film and the actual operation. Second, these differences are real—not due to chance variance, That differences exist is apparent

Results of Analysis of Variance on Ratings

Source	X^2	DF	V_E	F	SIG.
Between Stimuli Groups	503,838.7	175	2,879.0	289	.001
Within Stimuli Groups	157,994.7	1,584	99.7	—	—
Total	661,773.4	1,759	—	—	—

FIG. 1

from an examination of the raw data and from the means of the Groups for a given operator-job combination. The Analysis of Variance brings out very strongly that the differences are present through the extremely high variance estimate (VE). By testing this estimate in the "F" table we learn that there is considerably less than one chance in a thousand that the differences are due to chance. *We must therefore surmise that rating motion pictures is different from rating actual operations. The hypothesis is rejected.*

These differences may or may not have an effect on the use of film as it is presently employed. Further investigation into the situation would be necessary to definitely establish a predictable effect. Additional information can be gained from the 1840 observations of this experiment. Let us examine the raw data for more information.

Does the difference in rating materially affect the standard time? To test this hypothesis, the normal times

Results of Analysis of Variance on Normal Times

Source	X^2	DF	V_E	F	SIG.
Between Groups	41,545	167	.248772	712.81	.001
Within Groups	.528	1,512	.000349	—	—
Total	42.073	1,679	—	—	—
Between Stimuli*	.071	1	.071000	19.58	.001
Among Operators	.049	3	.016333	3.83	.050
Among Jobs	40.693	20	2.034650	390.75	.001
Operator-Stimuli Interaction	.003	3	.001000	1.52	N.S.
Stimuli-Job Interaction	.207	20	.010350	6.82	.001
Operator-Job Interaction	.431	60	.007183	4.74	.001
Triple Interaction	.091	60	.001517	4.35	.001

* Motion Pictures—Actual Operations

FIG. 2

were set for each rating and an analysis of variance was run to check differences. An average time for each job-operator combination was determined from the films. These were then multiplied by the rating of each observer. The resulting normal times to the nearest one-

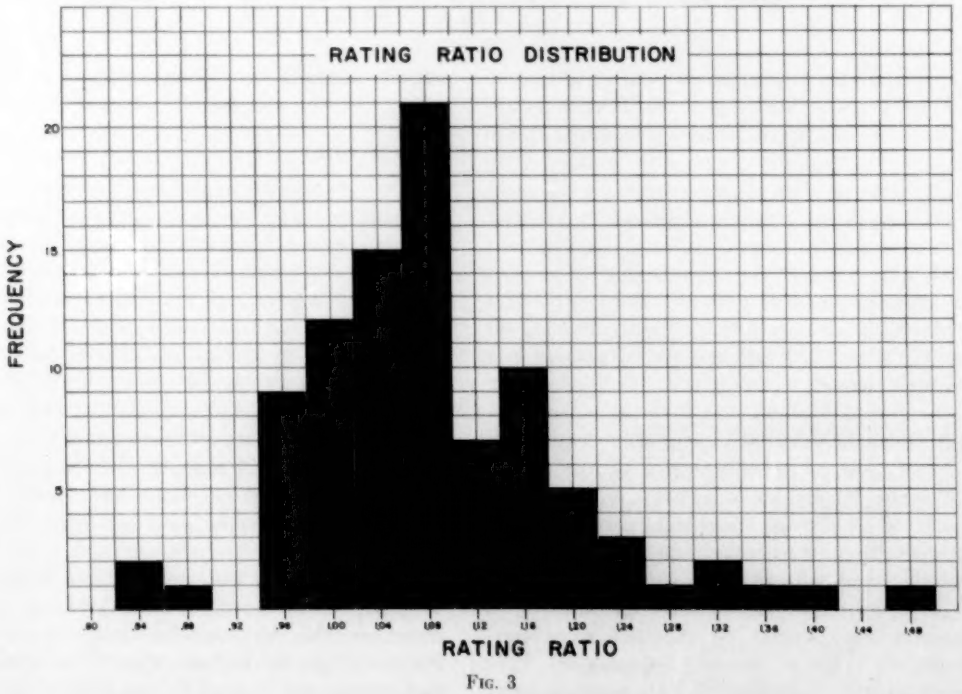

FIG. 3

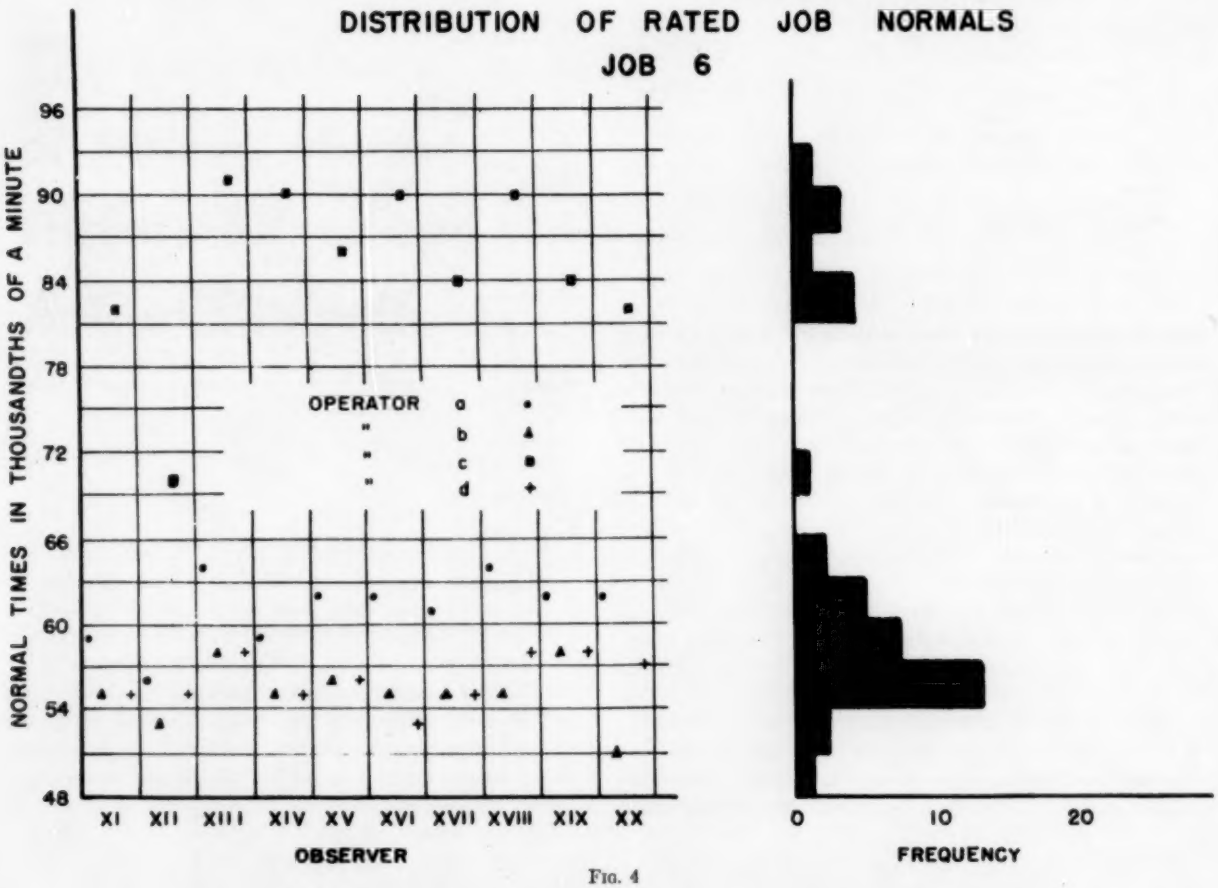

FIG. 4

thousandth of a minute was arranged similar to that of the original ratings. A complex triple classification analysis of variance was run which yielded the results shown in Figure 2.

From this test we see that *the differences are still present and highly significant* between the Stimuli, even after transformation to normal times.

How much difference are we encountering? The most meaningful expression of this would be in the form of a ratio or a percentage. Figure 3 is formed by dividing all film ratings by their corresponding actual ratings and plotting this in a frequency distribution. We immediately see that 1. the median is about 1.08, 2. the speed is excessive and 3. the distribution is positively skewed. All these factors together give us an informative picture of the situation. Certain factors seem to exist, although the films are rated on an average about eight percent higher, which influence the observer to frequently rate much higher than the eight percent. This may be associated with observer interactions which cause an observer to react differently than should be expected when confronted by certain types of jobs and/or operators. Further investigation will be required to learn more concerning the magnitude and behavior of the difference between ratings of film and actual operations.

CONCLUSIONS

These statements imply that an observer may rate job "1" performed by operator "a" different than he would the same job performed by operator "b" even though the two were equivalent. We examined the raw data to see if such a situation existed. Job 6 is an excellent example of this situation. In Figure 4 the normal times of job 6 are plotted so as to distinguish between all variables and then are projected into a frequency distribution. Operator "C" for job 6 is definitely rated differently than the other operators. A standard set on operators "a," "b" and "d" would be about .057 minute, whereas a standard set on operator "c" would be about .087 minute. It certainly makes a vast difference on which operator the observer sets the standard.

We should carry this just one step further and check the range of normal times for a given job for a group of observers. This was done for Group II and is shown in Figure 5. This table shows that 1. the observers cannot rate within the "expected" range of 5 percent, and 2.

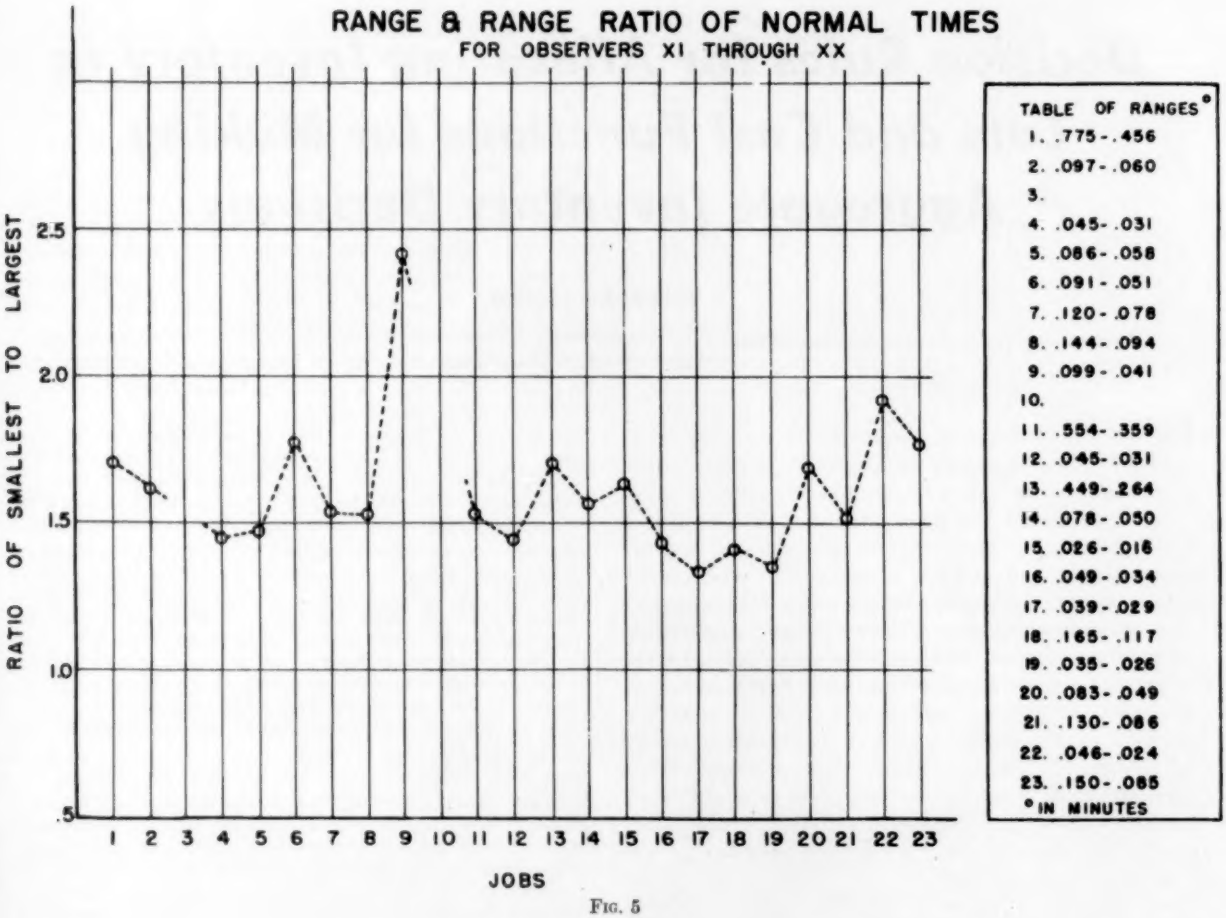

FIG. 5

there is a big difference in standards set by various observers on various operators. The first point might be disputed on the grounds that the group of raters used were not seasoned Time Study men. Therefore, they are not as consistent as is generally claimed for a group of Time Study men. This point was anticipated and steps were taken in the experiment to confirm or deny it.

At Plant II a group of seven Time Study men had a minimum of three years' experience within the plant. Each had an equal or greater amount of time study experience at plants other than Plant II. All had at one time or another made studies on and thereby set rates on all jobs within the plant. In short, if any group by contemporary standards could rate the jobs of Plant II with minimum variability, this group of Time Study men should.

They were given the same instructions as the students and were required to rate the actual operations and the films along with the students. The standard deviations of each group were computed. The students were *less* variable than were the Time Study men by approximately five percent. Therefore we feel that we were again justified in using the students as observers.

To conclude, we can state that 1. real differences do exist between the rating of film and actual operations, 2. films are rated on the average eight percent higher than actuals, 3. there are definite interactions between observers, operators, and stimuli.

Hudgins, Thompson, Ball and Associates

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Decision Rules for Allocating Inventory to Lots and Cost Functions for Making Aggregate Inventory Decisions[1]

by **CHARLES C. HOLT**

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INDUSTRIAL decisions frequently are made on different levels of an organizational hierarchy. At higher levels attention is given to broad considerations, and at lower levels more attention is given to minute details. This is an efficient way to structure complex decisions, and the application of quantitative techniques to business decisions should take this fact into account. This paper presents an analysis which is designed to facilitate decision-making on two levels. The analysis answers the following questions: If a decision has been made higher in the organization setting the aggregate amount of inventory available for lots (or batches), how should this inventory be allocated to lot sizes of individual products? What aggregate costs of set ups and aggregate costs of holding inventory should be taken into account by the higher level of the organization in setting the aggregate inventory that is to be available for lots without going into the detail decisions? To answer the first question decision rules are obtained for determining the optimal (purchase or production) lot sizes for individual products when a constraint has been placed on the total inventory that is permitted. To answer the second question, an expression is obtained for the total of the costs that depend on lot sizes, but this expression is given directly as a function of aggregate inventory. While this paper is written in the context of lot size and inventory decisions, the techniques developed should be of broad use in dealing quantitatively with complex multilevel decisions.

Simple formulas for determining optimal lot sizes for production or for purchases appeared in the literature in the nineteen twenties and have since been used by many companies. Franklin G. Moore asserts that their use is declining:

> Most companies today probably no longer compute economic-lot sizes. At the Hawthorne plant of the Western Electric Company . . . for example, a whole department was once engaged in computing economic lots. Today it has disappeared, and very little, if any, such computation is carried on. Perhaps part of the reason for the general decline in interest in economic lots has been the cost of computation. Furthermore, only approximate reliability of a computed answer can be counted on, since it is often impossible to forecast future needs and the possibility of obsolescence; also economic lots have only transitory validity because changes occur in demand, costs and other factors.
>
> General operational policies play an important part in actual lot size determination today. Quantities larger than the economic lot may be produced during temporary slack periods in order to level out production. Present and prospective price trends are important and play a part. The financial position of the company may limit inventory investment regardless of economic lots. Equipment limitations may force short runs to permit a variety of items to be produced on the machines available. Management may not know that production is being carried on in uneconomic lots or may not fully appreciate the costliness of uneconomic lots.
>
> If setting up is done by special setup men, the machine operators must be put on other work while the machine is being set up. Often work of equal caliber is not available, and the operator is idle or is used on lower grade work. Short runs cause extra costs in getting jigs, fixtures, or patterns from storage and returning them. These costs are rarely charged to the order. In some companies the accounting procedures charge setup costs also to an overhead account rather than to orders. This practice of charging machine accessory handling time and setup time to overhead accounts reduces the reported unit cost on short runs and tends to misinform management as to the costliness of uneconomic lots.
>
> Purchasing as well as manufacturing can be done in economic lots. Frequent orders for small quantities increase the cost of purchased items through increased clerical work, loss of quantity discounts, and higher transportation costs (5).

With electronic computers becoming more widely available to business firms, the computation of optimal lot sizes will no longer pose a serious problem. However, the other objections raised by Moore stand, and it is the opinion of the author that these come closer to the explanation of their limited use. Optimal lot size formulas are not "optimal enough," i.e., they do not take into account other relevant factors that are extremely important.

The present paper presents the results of an effort to remove some of the limitations of the traditional lot size analysis by considering the case in which an exact con-

[1] This paper is a shortened version of the original ONR Research Memorandum No. 37, Graduate School of Industrial Administration, Carnegie Institute of Technology. This research was undertaken for the project, *Planning and Control of Industrial Operations*, under Contract with the Office of Naval Research. Reproduction of this paper in whole or in part is permitted for any purpose of the United States Government.

straint is placed on aggregate inventory. Whether this constraint on aggregate inventory is set by a financial or production decision, by machine capacity, or by storage capacity will not be considered here (1) (2) (3) (4).

In general, individual products will each have different set up costs, different costs of holding inventory, and different sales rates. It would be desirable to take into account these characteristics for each product as well as the over-riding constraint that the aggregate inventory held as a result of lot size decisions to be at some specified level. Despite the complexity of the task of finding the interrelated decisions that minimize costs under these circumstances, the decisions should be reached without excessive decision making costs.

An exact solution that is general but computationally awkward is presented as well as a simple special case. By the use of quadratic approximations to the lot size cost functions a simple general solution is presented. The decision rules which are derived are used to aggregate the cost functions of the individual products so that total cost is obtained as a function of the aggregate inventory. This function may be used in making decisions which set the level of aggregate inventory.

In the following section we review the traditional lot size analysis.

I. LOT SIZE ANALYSIS FOR A SINGLE PRODUCT

We write an expression, Eq. 1., for the average cost per period of time for a lot size, Q, assuming that the sales rate, S, is known and constant for at least as long as a batch lasts.

$$C = \frac{C_F S}{Q} + \frac{C_I Q}{2} = C(Q, S) \qquad \text{Eq. 1.}$$

C = \$ Cost per period of time
C_F = \$ Set up cost for a lot (a constant)
S = Sales rate in units per period of time (an uncontrolled variable)
Q = Lot size in units of product (a controlled variable)
C_I = \$ Cost of holding one unit of inventory one period of time (a constant which should *not* include allocated fixed costs)
$Q^*(S)$ = Optimal lot size, unconstrained

Since at a sales rate, S, a lot of size, Q, will last a length of time, Q/S, if we divide the set up cost for producing or procuring a batch, C_F, by the above length of time, we obtain the average set up cost per period of time. In this way we obtain the first term of Eq. 1., $C_F/(Q/S)$.

Since in this analysis the inventory level fluctuates between zero and Q, and varies linearly in between (See Figure 1), the average inventory is $Q/2$. The cost of carrying this inventory is $C_I\, Q/2$, the second term of Eq. 1.

The total cost, C, is shown in Figure 2.

The first order condition for minimum cost is now obtained by equating to zero the slope of the total cost curve.

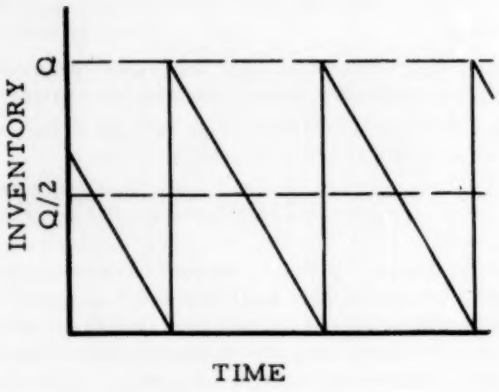

FIG. 1

$$\frac{\partial C}{\partial Q} = -\frac{C_F S}{Q^2} + \frac{C_I}{2} = 0 \qquad \text{Eq. 2.}$$

The value of Q that satisfies this relation is denoted as $Q^*(S)$, the optimum lot size for a product when there are no constraints on aggregate inventory. Solving for $Q^*(S)$ we obtain the familiar optimal lot size formula:

$$Q^*(S) = \sqrt{\frac{2C_F S}{C_I}} \qquad \text{Eq. 3.}$$

An application of this formula in obtaining minimum cost lot sizes for different sales rates is shown in Figure 2.

The above analysis of a single product can now be used in the analysis of many products by attaching the subscript, i, designating the ith product, to each of the coefficients and variables. The subscript, i, runs from 1 to N, the total number of products.

Whereas the foregoing analysis determines the lot size that minimizes the costs of set ups and inventory holding for a single product, we now turn to the problem of finding the lot sizes that minimize the total of these costs for all products subject to the constraint that a decision has

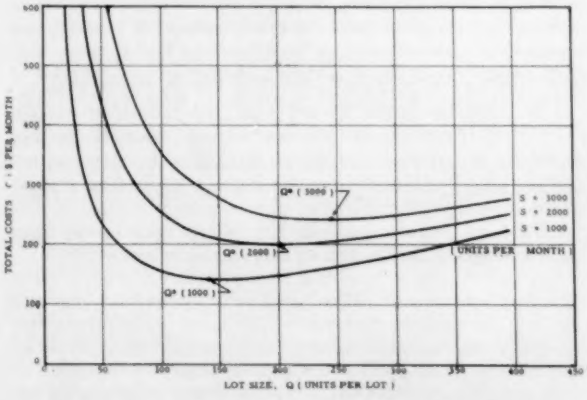

FIG. 2. Total Cost vs. Lot Size for Three Sales Rates

already been made setting the desired level of aggregate inventory.

II. LOT SIZE ANALYSIS FOR MULTIPLE PRODUCTS WITH A CONSTRAINT ON AGGREGATE INVENTORY

The cost to be minimized is now the sum of the costs associated with all of the individual products.

$$C_T = \sum_{i=1}^{N} C_i(Q_i, S_i) \qquad \text{Eq. 4.}$$

Where C_i is given by Eq. 1., the cost function is for an individual product. This total cost is to be minimized subject to the constraint that the total (aggregate) inventory of all products expressed in some common unit (for example, dollars, pounds, cubic feet, labor content, etc.) is I_Q, i.e.,

$$I_Q = \sum_{i}^{N} u_i \frac{Q_i}{2} \qquad \text{Eq. 5.}$$

Where:

u_i is the conversion factor for converting units of the ith product to the corresponding number of common units, and

$Q_i/2$ is the average inventory of the ith product, expressed in units of the product that is occasioned by its production or procurement in lots of size Q.

I_Q is the total inventory of all products in the common unit.

The aggregate inventory restraint may be either more or less than the sum of the optimal lot sizes for all products. For example, if a company were short of working capital, it might choose to produce in smaller than "optimal" lots, or if inventory were being built up in anticipation of high seasonal demand, a company might choose to produce in larger than "optimal" lots.

Strictly speaking Eq. 5. holds only on the average over a large number of products and in the steady state. That is to say, when I_Q is changed from one level to another and the lot sizes are changed accordingly, the actual inventory level will change to the new level with a distributed lag. The inventory held in half lots can be affected only by the actual production of new lots. The average lag in adjustment of the inventory level for an individual product is $Q_i/2S_i$ units of time.

An exact restraint relation can be stated in production terms, but this alternative case can be handled by the techniques presented here and will not be developed.[2]

The condition for minimum total cost is that the marginal cost of inventory is the same for all products. If this condition is not satisfied, total costs may be reduced by withdrawing a (common) unit of inventory from a high cost product and adding it to the inventory of a low cost product. This may be formalized by using the Lagrange multiplier to obtain the first order conditions for minimum costs:

$$C_T = \sum_i \left(\frac{C_{F_i} S_i}{Q_i} + \frac{C_{I_i} Q_i}{2} \right) + \lambda \left(\sum_i \frac{u_i Q_i}{2} - I_Q \right) \qquad \text{Eq. 6.}$$

where λ is the marginal cost of a common unit of inventory under optimal allocation.

$$\frac{\partial C_T}{\partial Q_i} = -\frac{C_{F_i} S_i}{Q_i^2} + \frac{C_{I_i}}{2} + \lambda \frac{u_i}{2} = 0$$

$$i = 1, 2, \cdots N. \qquad \text{Eq. 7.}$$

This expression may be solved for the constrained optimal lot sizes:

$$Q_i = + \sqrt{\frac{2C_{F_i} S_i / u_i}{\frac{C_{I_i}}{u_i} + \lambda}} \quad i = 1, 2, \cdots N. \qquad \text{Eq. 8.}$$

Note the similarity of this decision rule to Eq. 3. Substituting Eq. 8. in Eq. 6., we obtain:

$$I_Q = \sum_i + \sqrt{\frac{u_i C_{F_i} S_i / 2}{\frac{C_{I_i}}{u_i} + \lambda}} \qquad \text{Eq. 9.}$$

The structure of the solution can be seen most simply in a special case.

SOLUTION UNDER IDENTICAL INVENTORY HOLDING COSTS AND CONSTANT SALES COMPOSITION

If we make the not unreasonable assumption that the inventory holding cost of all items is the same when measured in the common unit, we may write:

$$\frac{C_{I_i}}{u_i} = C_{uI}, \text{ a constant}, \quad i = 1, 2, \cdots N. \qquad \text{Eq. 10.}$$

Thus from Eq. 9.

$$I_Q = \frac{1}{\sqrt{C_{uI}} + \lambda} \sum_i \sqrt{\frac{u_i C_{F_i} S_i}{2}} \qquad \text{Eq. 11.}$$

Solve for λ and substitute in Eq. 8. using Eq. 10., and we obtain the N decision rules for obtaining the constrained optimal lot size for each product.

$$Q_i = \left(2 \frac{\sqrt{\frac{C_{F_i} S_i}{u_i}}}{\sum_i \sqrt{u_i C_{F_i} S_i}} \right) I_Q \quad i = 1, 2, \cdots N \qquad \text{Eq. 12.}$$

If we assume further that the sales percentage composition is constant, i.e.

$S_i = K_i S$, where K_i is a constant, and S, the aggregate sales in the constant unit, is equal to $\sum_i u_i S_i$ and fluctuates over time. Eq. 13.

[2] Michael Gordon suggested that the restraint on lot sizes might come from the decision on aggregate production:

$$P = \sum_{i=1}^{N} n_i Q_i$$

where P is the aggregate production scheduled for a period of time, n_i is the number of lots of the ith product to be produced in the period. If production of the ith item is not scheduled for the period $n_i = 0$.

Then the decision rules simplify to the following expression which is free of the sales rates.

$$Q_i = \left(\frac{\sqrt{\frac{C_{Fi}K_i}{u_i}}}{\sum_i \sqrt{u_i C_{Fi} K_i}} \right) I_Q \quad i = 1, 2, \cdots N \qquad \text{Eq. 14.}$$

Since in this special case the bracketed expression is constant, the optimal allocation of aggregate inventory to lots of different products always gives the same percentage composition.

Having obtained decision rules that yield optimal lot size decisions, we may substitute these into Eq. 4. and Eq. 1. to obtain the minimum total cost under an inventory constraint, C_T min.

$$\min C_T = \left(\sum_i u_i C_{Fi} K_i \right) \frac{S}{2I_Q} + \frac{\left(\sum_i \frac{C_{Ii}}{u_i} \right) 2I_Q}{2} \qquad \text{Eq. 15.}^3$$

Comparison of this expression with the corresponding cost function for a single product, Eq. 1., reveals a striking parallel. The quantity $2I_Q$ corresponds to Q_i the factor 2 deriving from the fact that the average inventory level is $\frac{1}{2}$ the lot size. The constants in Eq. 15. are simply weighted sums of the cost coefficients of the individual products where the weights depend on u_i which converts the cost coefficients from the original inventory units to the common inventory unit, and K_i which indicates the relative sales importance of the individual products.

We now have obtained decision rules in Eq. 14. for allocating inventory over products given a higher level decision on aggregate inventory. And in Eq. 15. we have a cost function expressed in terms of aggregate sales rate and aggregate inventory which can be used in making the high level decision which sets the level of aggregate inventory.

One such analysis takes into account on the aggregate level the additional important costs of overtime cost, hiring, and laying off costs, and regular payroll costs (2) (3). In this analysis a straight forward calculation yields two simple linear decision rules, one for aggregate production and one for employment. Since aggregate production has direct implications for aggregate inventory, this decision can be used to determine the aggregate lot size inventory, I_Q, which in the foregoing discussion has been treated as a given constraint. Thus this analysis provides us with a method for determining the *optimal* constraint on the lot size decisions for the individual products. However, this simple solution depends on the special assumptions that the cost of holding inventory is the same for all products and the percentage sales composition is constant. The general case is not so neat.

[3] Using Eq. 10.

$$\sum_{i=1}^{N} \frac{C_{Ii}}{u_i}$$

may be simplified to NC_{uI}.

THE EXACT GENERAL SOLUTION FOR MULTIPLE PRODUCTS UNDER CONSTRAINT

Referring back to Eq. 9. before the special assumptions were introduced, we have a function of the form:

$$I_Q = f_{IQ}(\lambda) \qquad \text{Eq. 16.}$$

A simple analytic solution for λ as a function of I_Q does not appear to be possible, and hence a simple solution of this general case is blocked.[4] Fortunately, the lot size cost functions can be adequately approximated by other mathematical functions that yield simple solutions. One such is the quadratic function which when plotted is the quadratic function which when plotted is the familiar parabola. A further advantage of this particular function is that it leads to a decision rule that is a simple linear function, and it ties in readily with existing decision analyses on the aggregate level (2) (3). In the subsequent analysis the most general quadratic cost form will be used that is consistent with linear decision rules having constant coefficients. The next section develops methods for making this quadratic approximation. However as, yet no criteria have been developed for judging the optimality of these approximating methods.

[4] It is true that λ may be obtained as the root of a high order polynomial, but finding roots requires a resort to numerical methods. However, once we are resigned to the use of numerical methods, a still better method is available. (This approach to the cost aggregation problem was presented by Franco Modigliani in "Trigger Decision Rules for Allocating Inventory to Buffers and the Aggregation of Cost Functions." ONR Memorandum 47, Carnegie Institute of Technology.)

By numerical methods it is a straight forward task to calculate I_Q as a function of λ. To numerically evaluate I_Q for a particular λ requires the calculation of a sum composed of a large number of functions of λ. This whole process would then be repeated for other values of λ. Finally elementary numerical or graphical methods would be used to obtain the inverse function:

A) $$\lambda = f_\lambda(I_Q).$$

Since this function depends on the sales rates of all the individual products, it must be recomputed whenever there is a significant change in the sales composition. However, changes in the level of aggregate sales that do not involve changes in the sales composition may be handled without recomputation of the function.

The decision rules for obtaining the constrained optimal lot sizes for all the products involves the use of two equations. Starting with a value for I_Q, equation A) indicates the corresponding value of λ. This is then substituted into Eq. 8. to obtain the lot size for each product.

Now to obtain the aggregate cost corresponding to a value of I_Q, the optimal lot sizes are substituted into Eq. 4. and Eq. 1. Repeating this numerical computation for different values of I_Q yields a cost figure similar to Figure 2.

Computation of the exact aggregated cost function could be quite a large computing job unless it is possible to divide products into groups that each satisfy the conditions for the special case in which aggregation is simply performed, i.e., identical inventory holding costs and constant sales composition within the group. Each group could then be treated as an individual product thereby greatly reducing the required numerical computations.

III. METHODS FOR FITTING A QUADRATIC APPROXIMATION TO THE LOT SIZE COST FUNCTION

As shown by H. Theil the derivation of linear decision rules restricts us to cost functions having the following quadratic form (6):

$$C = \sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}X_iX_j + \sum_{i=1}^{n} f_i(Y_1, Y_2, \cdots)X_i + f(Y_1, Y_2, \cdots) \qquad \text{Eq. 17.}$$

Where: The X's are variables controlled by the decision maker and the Y's are uncontrolled variables. The C_{ij}'s must be constants, but the f functions may be of any form.

In the decision analysis partial derivatives are taken with respect to the controlled variables to obtain the conditions for minimum costs. These conditions are a set of simultaneous non-homogeneous linear equations in the controlled variables with additive terms in each equation that are functions of the uncontrolled variables. The coefficients of the square and cross product terms in the controlled variables appear in the matrix that multiplies the controlled variables. In order for this matrix to be constant so that it can be inverted once and for all, these C_{ij} coefficients must remain constant. There is no need for placing any restrictions on the other components of the cost function, and hence they may be any functions of the uncontrolled variables. The last term in Eq. 17., of course, is irrelevant for decision making.

In order to obtain cost functions that satisfy these requirements and approximate original cost functions, it is convenient to use a Taylor series expansion up through quadratic terms. The functions that are the coefficients of the square and cross product terms must then be replaced by C_{ij} constants that best approximate the function.

Two variations of this method are applied to the lot size cost function.

TAYLOR SERIES EXPANSION ON Q

If we take a Taylor series expansion of $C(Q, S)$ from Eq. 1. about the minimum cost point, $Q^*(S)$, which depends on the sales rate as shown in Eq. 3. and omit terms of higher order than square, we obtain the approximate cost by method A, i.e. C_A'':

$$C_A'' = C(Q^*, S) + \left(\frac{\partial C(Q, S)}{\partial Q}\bigg|_{Q=Q^*(S)}\right)(Q - Q^*) + 1/2\left(\frac{\partial^2 C(Q, S)}{\partial Q^2}\bigg|_{Q=Q^*(S)}\right)(Q - Q^*)^2 \qquad \text{Eq. 18.}$$

The terms of this cost function that are not affected by the controlled variables, Q, are irrelevant in making cost minimizing decisions, and hence may be dropped.

In view of the restriction above, that the coefficient of the squared term must be constant, we substitute for S it's average, $\overline{S}$, in that term. Although S is assumed constant in the short run, in the longer run it is assumed to vary in a forecastable way. For example, the sales rate may fluctuate seasonally, and although S varies, the seasonal average, $\overline{S}$, is a constant. We have assumed that the aggregate inventory restraint will cause the individual lot sizes to fluctuate about their "optimal" levels.

$$C_{A\,\text{error}}' = \left[\frac{\partial C(Q, S)}{\partial Q}\bigg|_{Q=Q^*(S)} - \frac{\partial^2 C(Q, S)}{\partial Q^2}\bigg|_{Q=Q^*(S)} Q^*\right] Q + \left[\frac{1/2\partial^2 C(Q, \overline{S})}{\partial Q^2}\bigg|_{Q=Q^*(\overline{S})}\right] Q^2 \qquad \text{Eq. 19.}$$

Evaluating the derivatives by reference to Eq. 1. and substituting the value of $Q^*(S)$ from Eq. 3., we obtain:

$$C_{A\,\text{error}} = -C_I Q + \left(\sqrt{\frac{C_I^3}{8C_F\overline{S}}}\right) Q^2 \qquad \text{Eq. 20.}$$

Note that in this expression the sales forecast variable, S, has cancelled out, thus indicating a poor approximation to Eq. 1. in which there is an important cost interaction between S and Q. This case is presented in order to draw attention to a serious error that might go unnoticed in a more complex derivation. The error, which was made in going from Eq. 18. to Eq. 19. involves a violation of the premises of the Taylor series expansion. The Taylor expansion of Eq. 18. decomposes the function into additive level, linear, and curvature components. When we restricted the curvature coefficient,

$$\left(\frac{\partial^2 C(Q, S)}{\partial Q^2}\bigg|_{Q=Q^*(S)}\right),$$

to constancy for the Q^2 term of Eq. 19., we should have applied the same value to the cross product term, QQ^*. From Eq. 18. it is clear that the curvature term depends on the *difference* between Q and Q^*; hence it is essential that the same curvature coefficient apply to all of the terms arising from the expansion of the expression, $(Q-Q^*)^2$.

Instead of Eq. 19. we should have obtained:

$$C_A' = \frac{\partial C(Q, S)}{\partial Q}\bigg|_{Q=Q^*(S)} + 1/2\left(\frac{\partial^2 C(Q, \overline{S})}{\partial Q^2}\bigg|_{Q=Q^*(\overline{S})}\right)(Q - Q^*)^2 \qquad \text{Eq. 21.}$$

Evaluating the derivatives and substituting for $Q^*(S)$ as before we obtain:

$$C_A = \left(\sqrt{\frac{C_I^3}{8C_F\overline{S}}}\right)(Q - Q^*(S))^2 = \left(\sqrt{\frac{C_I^3}{8C_F\overline{S}}}\right)\left(Q - \sqrt{\frac{2C_F\overline{S}}{C_I}}\right)^2 \qquad \text{Eq. 22.}$$

This is an eminently reasonable result in indicating that costs rise as Q deviates from Q^* in either direction, Q^* being determined by the optimal lot size formula as a function of sales.

A refinement which might be useful but which would cost something in computation would be to substitute

$$\sqrt{\frac{1}{S}} \quad \text{for} \quad \sqrt{\frac{1}{\overline{S}}}$$

in determining the constant for the square term, i.e., use the average of the square root of the reciprocal instead of the square root of the reciprocal of the average.

A basic limitation of this approximation is that the rate at which costs rise on either side of $Q^*(S)$ does not change with changes in the sales, but is constant. However, this error should not be serious since ideally the rate should change only as the square root of sales.

TAYLOR SERIES EXPANSION ON Q AND S

Should it be desired to avoid the computation of the square root of S by using its linear approximation in determining Q^*, another method is available for fitting the quadratic. If we make a Taylor series approximation of Eq. 1., in the two variables S and Q about the constant point $\overline{S}$ and $Q^*(\overline{S})$ and drop all terms of higher order than quadratic, we obtain the approximate cost by method B, i.e., C_B'':

$$C_B'' = C[Q^*(\overline{S}), \overline{S}] + \left[\frac{\partial C(Q,S)}{\partial Q}\bigg|_{\substack{Q=Q^*(\overline{S})\\S=\overline{S}}}\right][Q - Q^*(\overline{S})]$$

$$+ \left[\frac{\partial C(Q,S)}{\partial S}\bigg|_{\substack{Q=Q^*(\overline{S})\\S=\overline{S}}}\right][S - \overline{S}]$$

$$+ 1/2\left[\frac{\partial^2 f(Q,S)}{\partial Q^2}\bigg|_{\substack{Q=Q^*(\overline{S})\\S=\overline{S}}}\right][Q - Q^*(\overline{S})]^2$$

$$+ 1/2\left[\frac{\partial^2 C(Q,S)}{\partial S^2}\bigg|_{\substack{Q=Q^*(\overline{S})\\S=\overline{S}}}\right][S - \overline{S}]^2$$

$$+ \left[\frac{\partial^2 C(Q,S)}{\partial Q \partial S}\bigg|_{\substack{Q=Q^*(\overline{S})\\S=\overline{S}}}\right][Q - Q^*(\overline{S})][S - \overline{S}] \quad \text{Eq. 23.}$$

Now drop those components that do not involve Q, and hence are irrelevant for decision making.

Carrying out the differentiation and completing the square by the addition of a function of S which, of course, is irrelevant to the decision, we obtain:[5]

$$C_B = \sqrt{\frac{C_I{}^3}{8C_F\overline{S}}}\left[Q - Q^*(\overline{S})\left(\frac{1 + S/\overline{S}}{2}\right)\right]^2 \quad \text{Eq. 24.}$$

[5] There is no necessity for completing the quadratic to the perfect square form, but it facilitates comparison with the result attained above in giving an expression for the minimum point of the quadratic.

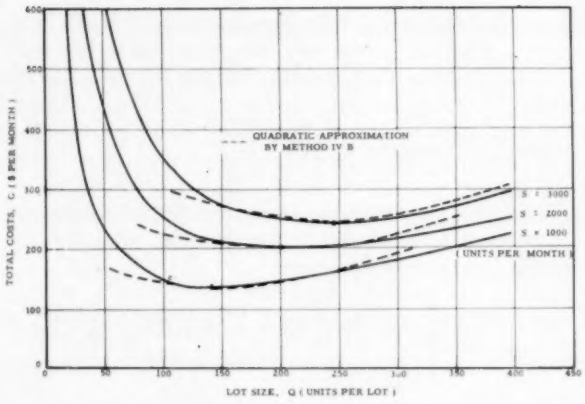

FIG. 3. Total Cost vs. Lot Size Actual and Approximated

The minimum point of the quadratic is the optimal lot size for the average sales rate modified by a linear, and hence the easily computable, function of the sales rate.[6]

This approximation is applied to the particular cost coefficients and sales rates presented in Figure 2. The results are presented in Figure 3. Since the approximating function has dropped the cost components that depend only on sales rate because they are irrelevant in making lot size decisions, it is necessary to reintroduce these terms in order to make comparisons of total costs.

IV. AN APPROXIMATE GENERAL ANALYSIS FOR MULTIPLE LOTS UNDER CONSTRAINT

In this section we will show that if for each product the cost function of lot size dependent costs is approximated by a quadratic function of the form used in the preceding section, we can obtain without any restrictions on generality the following:[7]

1. The approximate constrained optimum lot sizes for each product by means of simple decision rules.
2. An approximate aggregated cost function of the desired quadratic form expressed in terms of aggregate inventory.

If for each product we approximate its cost function[8] from Eq. 1. by the quadratic fit of Eq. 22. and substitute the approximate cost function in the expression for total

[6] Since the series expansion was carried only through square terms, the highest order interaction allowed between Q and S was a linear relation.

[7] Robert F. Byrne worked with the author in developing this approach.

[8] The costs of holding inventory that are proportional to the common inventory unit prove to be irrelevant to the allocation of the constrained aggregate inventory. Hence, there is an administrative economy in not allocating these costs over the individual products. However, in the aggregate these costs are important so if they are dropped from the C_{Ii} coefficients, they should be added back in Eq. 4., 15., 25. and 29. by including a term of the form $C_I'I_Q$. Also an adjustment may be required in the points at which the Taylor series approximation are fitted. These points are $Q_i^*(S)$ which depend on the cost of holding inventory and hence are sensitive to dropping an important cost component from the C_{Ii}'s.

cost, Eq. 4. we obtain the approximate total cost, C_{AT}:

$$C_{AT} = \sum_{i=1}^{N} C_{Qi}[Q_i - Q_i^*(S_i)]^2 \qquad \text{Eq. 25.}$$

where C_{Qi}, a constant $= \sqrt{\dfrac{C_{Ii}^2}{8C_{Fi}\overline{S}_i}}$

We derive the first order conditions for minimum cost as in Section II to obtain the decision rules for the constrained optimal lot sizes:

$$Q_i = \left[\frac{u_i}{C_{Qi}}\right]\left[\frac{2}{\sum_i \dfrac{u_i^2}{C_{Qi}}}\right]\left[I_Q - \sum_i u_i Q_i^*(S_i)/2\right] + Q_i^*(S_i) \qquad i = 1, 2, \cdots N. \qquad \text{Eq. 26.}$$

Each of these decision rules very reasonably calls for the "optimal" lot size plus or minus a fraction of the deviation between aggregate inventory and the sum of the "optimal" half lots expressed in the common unit. In calculating this sum little error will be introduced by assuming that the sum of the optimal lot sizes is not affected by changes in the percentage sales composition, i.e.:

$$\sum_i u_i Q_i^*(S_i) \approx \sum_i u_i Q_i^*(K_i S) \qquad \text{Eq. 27.}$$

Where: K_i is the constant, $\dfrac{\overline{S}_i}{\overline{S}}$

S is the aggregate sales rate expressed in the common unit, $\sum_{i=1}^{N} U_i S_i$.

$\overline{S}_i$ and $\overline{S}$ are the average values forecasted for S_i and S respectively. The relevant forecast horizon is the period during which the decision rules will not be revised. Substituting Eq. 3. and Eq. 27. in Eq. 26. we obtain the decision rules for constrained optimal lot sizes in terms of cost parameters and sales forecasts:

$$Q_i = \left[\frac{u_i}{C_{Qi}}\right]\left[\frac{2}{\sum_i \dfrac{u_i^2}{C_{Qi}}}\right]\left[I_Q - \left(\sum_i u_i \sqrt{\frac{C_{Fi}\overline{S}_i}{2C_{Ii}\overline{S}}}\right)\sqrt{S}\right] + \sqrt{\frac{2C_{Fi}S_i}{C_{Ii}}} \qquad i = 1, 2, \cdots N. \qquad \text{Eq. 28.}$$

Note that the decision for any lot size involves not only a sales (or production usage) forecast of the particular product, but also a forecast of aggregate sales (or production usage) and the aggregate inventory that is available for lots. While these decision rules are approximate in that a quadratic approximation was used in their derivation, they are general in that no restrictions are placed on costs and only a very mild restriction on the sales composition. These decision rules can be calculated once, and as long as these costs are unchanged and the sales composition does not change drastically, these rules can be used under conditions of varying product and aggregate sales forecasts, and different aggregate inventory levels. However, they should be recalculated when the cost coefficients or the long term forecasts of average sales for individual products have changed significantly. These rules will efficiently allocate inventory taking into account the restriction of the higher level decision on aggregate inventory level.

If we substitute these decision rules into Eq. 25. to obtain the minimum total cost, under the inventory restraint, we obtain:

$$C_{AT_{\min}} = \left[\frac{4}{\sum_i \dfrac{u_i^2}{C_{Qi}}}\right]\left[I_Q - \left(\sum_i u_i \sqrt{\frac{C_{Fi}\overline{S}_i}{2C_{Ii}\overline{S}}}\right)\sqrt{S}\right]^2 \qquad \text{Eq. 29.}$$

This total cost function of the two aggregate variables, inventory and sales, has suppressed the detail decisions that are implicit in it. Also all costs that are not affected by the inventory decision have been dropped. Hence this cost function goes to zero when the aggregate inventory is set at the level equal to the sum of all the "optimal" half lots. The coefficients of this cost function would have to be recomputed when the cost coefficients or long term sales forecasts changed significantly. However, the cost function would be nowhere near as sensitive to these changes as would the decision rules for individual products.

The aggregated cost function that has been obtained may now be used in conjunction with hiring, lay off, overtime and other costs in making the aggregate production decision that determines aggregate inventory. This in turn may be used to determine the constrained optimal lot size for each product.

Sudden changes in the *desired* aggregate lot size inventory can quickly be translated into decisions to change the product lot sizes. However, since for any particular product there may be some time before it comes due for the production of another lot, it is evident that there is a distributed lag in actually achieving the new level of aggregate lot size inventory that is desired.

Treating the sales rate as a random variable in the case of uncertainty introduces no serious problems as has been shown in part by Franco Modigliani and John F. Muth (7). This may be seen in part from Eq. 1. where it is clear that $E[C(S)] = C(ES)$. Thus the foregoing analysis can be modified to minimize the mathematical expectation of cost simply by interpreting S as an expected value forecast, i.e., the mean of the sales rate distribution. The mean sales rate may change, seasonably for example, hence we need to fit the Taylor series at a point corresponding to the average of the expected sales rates. Thus $\overline{S}$ should be interpreted as the average of the expected sales rates.

In summary a numerical example will show how a practical application of this analysis might be made. Eq. 28., Eq. 29. and the following definitions constitute the essential results.

$$S \equiv \sum_{i=1}^{N} u_i S_i, \quad \overline{S} \equiv \sum_{i=1}^{N} u_i \overline{S}_i, \quad C_{Qi} \equiv \sqrt{\frac{C_{Ii}{}^3}{8 C_{Fi} \overline{S}_i}} \qquad \text{Eq. 30.}$$

Assume that in an operating situation the estimates of the set up costs and the costs of holding inventory for each product are reviewed and revised annually. When this is done, forecasts are also made for each product of the *average* monthly sales rate, $\overline{S}_i$, that is expected for the coming year. The aggregate inventory control decision is made in terms of the labor content of the products, i.e., changes in the inventory level are planned with a view to the labor requirements as well as inventory costs. Thus, to obtain aggregate inventory, the units of each product are converted to labor hours by multiplying by the u_i conversion factor, labor hours per unit of product. This information is tabulated for a factory producing three products:

Product Number	Cost Coefficients		Conversion Factors
	C_{Ii}	C_{Fi}	u_i
$i=1$	1 \$/units of product 1×mo.	10 \$	1 hr /unit of product 1
$i=2$	1 \$/units of product 2×mo.	10 \$	5 hrs/unit of product 2
$i=3$	2 \$/units of product 3×mo.	30 \$	2 hrs/unit of product 3

Product Number	Annual Product Sales Forecasts $\overline{S}_i$
$i=1$	2,000 units of product 1/mo.
$i=2$	2,000 units of product 2/mo.
$i=3$	10,000 units of product 3/mo.

The aggregate sales forecast for the year, $\overline{S}$, is (1×2,000)+(5×2,000)+(2×10,000) or 32,000 hrs. of product/mo.

One consideration in setting the aggregate inventory level is the total inventory holding and set up costs that are associated with each level of aggregate inventory. Substitution of the foregoing cost coefficients, conversion factors, and annual sales forecasts into Eq. 29. yields a cost function for making quantitative estimates of the above mentioned costs:

$$\underset{\text{min}}{C_{AT}} = .000317(I_Q - 6.42\sqrt{\overline{S}})^2 \quad \text{\$/mo.} \qquad \text{Eq. 31.}$$

These costs depend on the interaction between the expected level of aggregate sales and the aggregate inventory level.[9] For example, if aggregate sales rise without an increase in aggregate inventory, then clearly the set up costs will increase. The foregoing cost function together with other cost functions would be used in making monthly decisions on the most desirable level for aggregate inventory, I_Q. This function would be used for a year before it would need to be recomputed.

Substituting the aforementioned cost coefficients and long range sales forecasts into Eq. 28., we obtain (to slide rule accuracy) the decision rules that will be used throughout the year for computing the lot sizes for each product.

$$\begin{aligned} Q_1 &= .0636(I_Q - 6.42\sqrt{\overline{S}}) + 4.48\sqrt{\overline{S}_1} \\ Q_2 &= .318\ (I_Q - 6.42\sqrt{\overline{S}}) + 4.48\sqrt{\overline{S}_2} \\ Q_3 &= .174\ (I_Q - 6.42\sqrt{\overline{S}}) + 5.48\sqrt{\overline{S}_3} \end{aligned} \qquad \text{Eq. 32.}$$

In these decision rules lot sizes depend not only on the product sales forecasts, S_i, as in simple lot size formulas, but depend also on the aggregate sales forecast, S, and the aggregate inventory, I_Q, that is set by the higher level decision.

Assuming that lot sizes are adjusted monthly, the following represent a typical set of forecasts at the beginning of a particular month.

Monthly Sales Forecast S_i
S_1 = 2,500 units of product 1/mo.
S_2 = 2,500 units of product 2/mo.
S_3 = 8,000 units of product 3/mo.

The aggregate monthly sales forecast, S, is (1×2,500) +(5×2,500)+(2×8,000) or 26,000 hours of product/mo.

A decision setting the aggregate inventory level, I_Q, equal to 1,300 hours of product would then yield the following lot sizes using Eq. 32.[10]

$$\begin{aligned} Q_1 &= 241 \text{ units of product 1} \\ Q_2 &= 308 \text{ units of product 2} \\ Q_3 &= 536 \text{ units of product 3} \end{aligned}$$

It is interesting to observe that the percentage composition of inventory by product may be significantly different at different levels of aggregate inventory. This may be shown by using the foregoing monthly sales forecasts and calculating the lot sizes for three different aggregate inventory levels. By converting these lot sizes to the common labor hour unit, the percentage inventory composition is obtained, shown at top of next page.

Since the methods developed in Sections III and IV are approximate, it is pertinent to inquire how these results would compare with the exact methods developed in Section II. If the following numerical example is used,

[9] Cost function 31 includes only those cost components that are affected by the controlled variable, I_Q. These are the only costs that are relevant to the decision setting the level for I_Q. A zero value for this cost function simply means that *these* cost components are at their minimum. Clearly it does not mean that total set up and inventory holding costs are zero.

[10] In this small sample of products the simplifying assumption of Eq. 27. is not adequately satisfied with the consequence that the half lots when converted to the common unit, labor hours, do not exactly add up to the aggregate inventory constraint, I_Q. The use of Eq. 26. instead of Eq. 28. would have avoided this difficulty. However, in a large sample of products this discrepancy would probably be of negligible size.

Inventory Composition By Product $\frac{u_i Q_i}{2I_Q}$	Aggregate Inventory Constraint		
	$I_Q = 800$	$I_Q = 1{,}035$	$I_Q = 1{,}300$
Product 1	11.3%	9.7%	8.5%
Product 2	40.4	48.2	53.0
Product 3	48.3	42.1	38.5
Aggregate	100%	100%	100%

solutions for λ can be obtained from Eq. 9. using numerical methods. Substitutions in Eq. 8. will then yield the exact constrained optimum lot sizes. Using the same aggregate inventory for the exact and approximate solutions we obtain the following results (to slide rule accuracy):

Aggregate Inventory

"Optimum" Lot Sizes	$I_Q = 927$		$I_Q = 1162$		$I_Q = 1427$	
	Exact	Approximate	Exact	Approximate	Exact	Approximate
$Q_1 =$	202	209	225	224	236	241
$Q^2 =$	146	149	218	224	317	308
$Q_3 =$	425	449	473	490	497	536

The lot size decisions yielded by the approximate and computationally simple method are close to the lot sizes given by the exact method, although there are noticeable differences. However, when these lot size decisions are substituted in Eq. 4. to obtain the actual cost performances, the cost difference between the exact and approximate decisions is smaller than the slide rule error. A less simplified example and greater computational accuracy would be required to adequately evaluate the slight cost increase attributable to using the approximate method.

It seems clear that the methods proposed are better than simple lot size formulas for situations in which aggregate constraints are important, and they are probably accurate enough for practical purposes when account is taken of the computing cost of further refinement. The foregoing calculations were made with the decision rule of Eq. 28.; greater accuracy (especially in satisfying the aggregate constraint) can readily be obtained by using Eq. 26. instead.

In conclusion, the point should be reemphasized that this analysis was designed to facilitate two level decision making. While it is useful for lot size problems, it is not limited to them. The combination of mathematical tools that have been used here should apply equally well to many other decision problems including those that involve multiple aggregate constraints.[11] The decision results that have been achieved are simple enough to be performed by clerical computation even for a large number of products and, of course, could be very easily performed on an electronic computer.

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[11] Peter R. Winters in "A Trigger Decision Rule for Allocating Inventory to Production Lots and Buffers" (ONR Research Memorandum No. 48, Carnegie Institute of Technology) has applied this approach simultaneously to the determination of production lots and buffers.

For a similar analysis not using the quadratic approximation, see "Trigger Decision Rules for Allocating Inventory to Buffers, and the Aggregation of Cost Functions," by Franco Modigliani and Sharu S. Rangenekar (ONR Memorandum No. 47, Carnegie Institute of Technology).

New Organization Analysis Techniques[1]

by **IRWIN P. LAZARUS**[2]

Director, Ordnance Management Engineering Training Program
Vice-President, Midwest Region, AIIE, 1956-57

EVERYONE feels he is an organization expert. Even you feel this way and, if you are truly convinced, please read no further. I believe one of the reasons everyone is so sure that he is an expert in the field of organization is that man has been organizing nearly as long as he has been man. It has been said that a continuing sex drive led early man to the organization of the family. Fear led him to the organization of the tribe. Finally, economic motivation or greed led man to the development of industrial organizations.

If we are interested in organization science as it has existed, we must turn to the military for our greatest wealth of written material and theoretical development. The first recorded organization structure was drawn by a Pharaoh of Egypt approximately 3000 B.C. prior to an attack on what we now call Syria. He recorded the organization of his troops to serve as a model for many centuries. Industrial organization structure can be traced to the Industrial Revolution, or had its beginning approximately 1770 A.D. Only when the family artisans were brought together under the factory roof did it become necessary to draw up organization patterns to best accomplish the work. Thus, industrial organization followed military organization by about 4,700 years.

As a further indication of the lag of industrial organization behind military organization, in the year 1763, Frederick the Great of Prussia developed the General Staff concept in order to better manage the technological complexities that the Prussian Army was embracing. Only recently, beginning in the early 1950's, some of the large American organizations such as General Motors and Westinghouse began moving toward the General Staff concept in order to best handle the complex technical problems with which they were faced. So much for the past. But one might ask himself, "Is there anything really new under the sun in the area of organization analysis?"

LINEAR RESPONSIBILITY CHART

The first organization analysis technique we tried at the Ordnance Management Engineering Training Program was originally developed by a Netherlands consulting engineer named Ernst Hijmans and was brought to this country by Mr. Serge Birn, an American consultant who met the Hollander in Europe. This technique is called the Linear Responsibility Chart and was described by Mr. Birn in *Dun's Review and Modern Industry*, September 1954, pp. 46-49. The Linear Responsibility Chart is a graphic picture of the Organization Manual. Across the top of the chart are listed the segments of the organization that one is interested in analyzing. Up the left side of the chart are listed the functions that are under analysis. In the body of the chart are drawn graphic symbols to depict the amount of authority and responsibility that each organizational segment exercises over the functions in question. These eight graphic symbols vary from the actual doing of the work to calling a person in for an exchange of views. On the example chart, shown as Figure 1, the number of persons exercising authority and responsibility over the functions in question have been reduced from 42 total steps to 21 total steps. It should be remembered that the source for the Linear Responsibility Chart is the Organization Manual.

It was felt by the staff of the Ordnance Management Engineering Training Program that the Linear Responsibility Chart, while visually describing the Organization Manual in a much clearer fashion than the manual did itself, still tended to confuse the user. It was felt that the shaded and crosshatched symbols tended to flow together and make objective analysis difficult.

MULTIPLE CORRELATION CHART

In order to compensate for the difficulty of understanding the maze of slightly different graphic symbols, our staff developed what it calls the Multiple Correlation Chart for Organization Analysis. The Multiple Correlation Chart for Organization Analysis has one basic similarity with the Linear Responsibility Chart. This is the layout of the chart in which the segments of the organization are placed across the top of the chart and the functions undergoing analysis are placed vertically along the left side of the chart. Underneath each segment of the organization and across from the function in question is placed the amount of authority and responsibility that each organizational group exercises. However, instead of using graphic symbols, the Multiple Correlation Chart is made by cutting from the source document the actual

[1] Based upon a presentation to the Sixth Annual Conference of the Cleveland Chapter, AIIE, Cleveland, Ohio, March 28, 1957.

[2] Presently Associate Director, Central Management Staff, American Colortype Company, Chicago, Illinois.

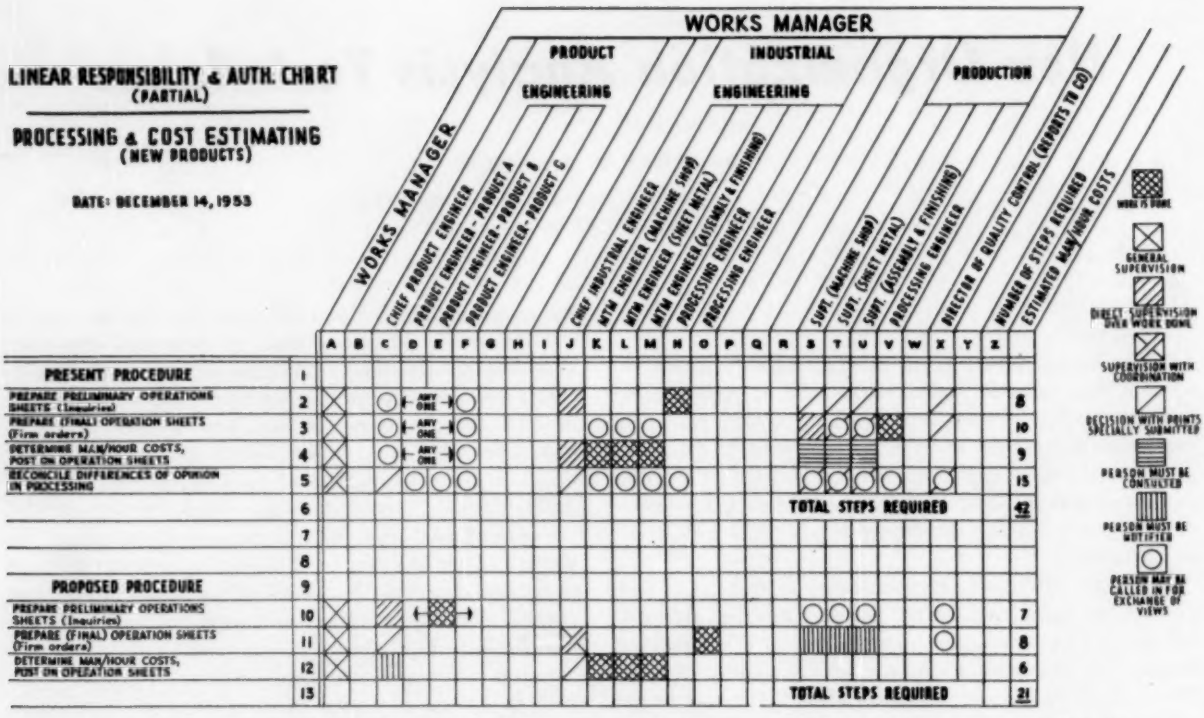

FIG. 1. Before-and-after Charts, Disguised, but from A Real Plant, Show How Organizational Faults Are Detected, Remedied—("Reprinted with permission from *Dun's Review and Modern Industry*")

sentences or paragraphs that describe the amount of authority and responsibility that each organizational segment exercises over the function under investigation. Furthermore, instead of using only the Organization Manual as a source document, the Multiple Correlation Chart is made from five source documents. These source documents are:

1. The Organizational Manual.
2. Job Descriptions.
3. Standard Procedures and Method Descriptions.
4. Policy Statements.
5. Actual Operations.

After snipping the statements from the manual, from job descriptions, etc., and pasting them on the Multiple Correlation Chart, the analyst follows the pasting with a color-coding to indicate which of the five sources caused the statement that was pasted to the chart. The analyst either encircles or underlines the paragraph with the color describing the source. In other words, if the information was snipped from the Organization Manual that paragraph will be encircled in red. The job descriptions will be encircled in brown. Procedures and methods are encircled in orange. Policy statements are encircled in blue and the actual operations are encircled in green. An example of the Multiple Correlation Chart for Organization Analysis appears as Figure 2.

Following the construction of the Multiple Correlation Chart, an examination is made to locate conflicts of authority and responsibility exercised by the various segments of the organization. As can be seen from Figure 2, a conflict exists between the Deputy Chief of Ordnance and the Commanding General, OWC, in the function of mobilization planning and this conflict is depicted with the number 1 encircled. Other conflicts are shown as numbers 2, 3, 4, etc. After all conflicts are located, the chart can be brought into the office of the deciding executive and a reconciliation can be made to determine who should have the final authority and responsibility for the function in question.

SPECTRUM ANALYSIS

Our staff has used the Multiple Correlation Chart a number of times and often interesting problems have arisen when some of the source documents were missing. For instance, an organization recently requested our assistance to make an organization study of their offices since they felt they did not have the time or the objectivity to do the job themselves. We began by gathering the data necessary to make the Multiple Correlation Chart. When we requested their Organization Manual, they informed us that they had none. When we requested their Procedure Manual, they informed us that they had not yet gotten around to writing one. When they showed us their job descriptions, we found that the job descriptions were worded so generally and in such high-sounding terms as to be utterly useless for our purpose. We were

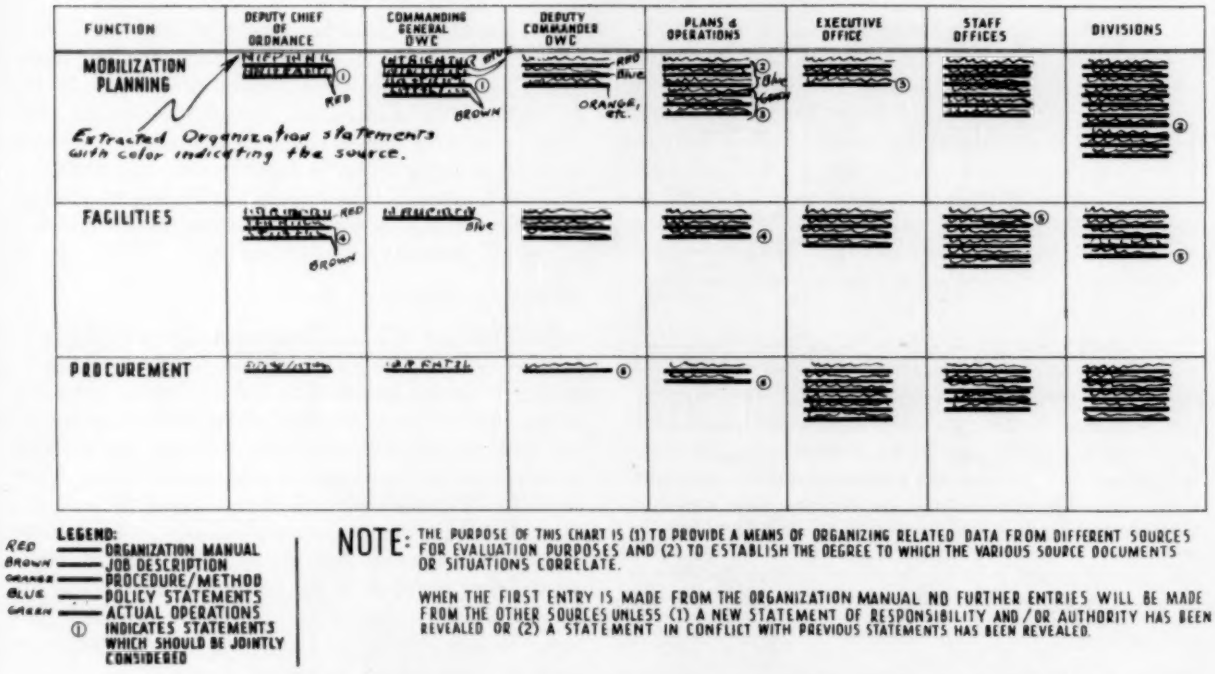
FUNCTION
DEPUTY CHIEF OF ORDNANCE
COMMANDING GENERAL DWC
DEPUTY COMMANDER DWC
PLANS & OPERATIONS
EXECUTIVE OFFICE
STAFF OFFICES
DIVISIONS
MOBILIZATION PLANNING
FACILITIES
PROCUREMENT
Extracted Organization statements with color indicating the source.
RED
BROWN
BLUE
ORANGE, etc.
LEGEND:
RED —— ORGANIZATION MANUAL
BROWN —— JOB DESCRIPTION
ORANGE —— PROCEDURE/METHOD
BLUE —— POLICY STATEMENTS
GREEN —— ACTUAL OPERATIONS
① INDICATES STATEMENTS WHICH SHOULD BE JOINTLY CONSIDERED
NOTE: THE PURPOSE OF THIS CHART IS (1) TO PROVIDE A MEANS OF ORGANIZING RELATED DATA FROM DIFFERENT SOURCES FOR EVALUATION PURPOSES AND (2) TO ESTABLISH THE DEGREE TO WHICH THE VARIOUS SOURCE DOCUMENTS OR SITUATIONS CORRELATE.
WHEN THE FIRST ENTRY IS MADE FROM THE ORGANIZATION MANUAL NO FURTHER ENTRIES WILL BE MADE FROM THE OTHER SOURCES UNLESS (1) A NEW STATEMENT OF RESPONSIBILITY AND/OR AUTHORITY HAS BEEN REVEALED OR (2) A STATEMENT IN CONFLICT WITH PREVIOUS STATEMENTS HAS BEEN REVEALED.

FIG. 2. Multiple Correlation Chart for Organization Analysis (Using a Linear Presentation of the Responsibility & Authority Pattern of the Planning Function)

left with two sources. These were policy statements in the form of regulations and actual operations as described by the individuals in the organization.

We took these remaining two sources available to us and utilized them as best we could. A large card was devoted to each segment of the organization. On the card were pasted statements describing the organization's important activities. These descriptions were cut from the various regulations pertaining to the organizational segment. After interviewing the personnel of the organizational segment described on the card, the actual method of performance was determined. This was then shown by attaching colored cellophane directly over the printed matter; with the color of the cellophane coded to represent the method of performance. The cellophane was a wax-backed product commonly used for making charts and graphs (it can be purchased under the trade-name of Zip-A-Tone). The color-coding was as follows: If the organization was not performing the activity described by the policy statement, a red cellophane covered up the words and indicated those words were "not being performed." (The words could be very easily read through the colored cellophane it should be remembered.) If the activity under question was being performed elsewhere, a yellow cellophane overlay was used. If the activity was sometimes performed by the organization, sometimes not performed by the organization, and generally highly questionable in the minds of the personnel, a gray overlay was used to depict a gray area. The analysis team had originally planned to use four more colors and the addition of these colors would give a more detailed analysis of the activities in question. These are:

1. Green—Not being performed adequately.
2. Purple—Being performed in too great detail.
3. Blue—Being performed but the activity was not assigned.
4. Brown—Being performed but this activity *should not* have been assigned.

Along side of the cellophane overlays, the group wrote a recommendation for each situation. In the case of the red cellophane, a typical recommendation was that staff be secured to do the job adequately. In the case of the yellow cellophane (meaning that the work was performed elsewhere), it was often recommended that the work remain elsewhere since it was being performed adequately. This chart clearly indicated deficiencies in the performance of the organization and was highly effective in gaining top management attention. The technique has since been titled the Spectrum Analysis and has proved to be highly effective.

INFORMAL ORGANIZATION CHART

Another technique which has proved to be highly effective in describing the activities of an organization to its chief was also utilized in the study mentioned above. This is the device known as the Informal Organization Chart. The Informal Organization Chart was drawn by placing the various organizational boxes in the same relative position as on the Formal Organization Chart.

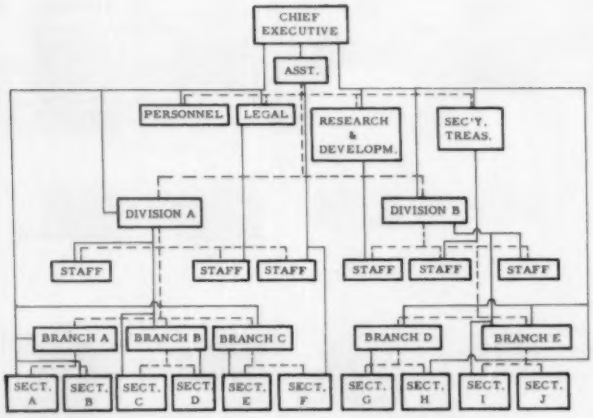

FIG. 3. The Informal Organization Chart. Dash lines signify formal lines of supposed authority and responsibility. Solid lines signify actual lines of habitual reporting and directing. Note individuals reporting two levels above rather than to their immediate superior. Also, thirteen people report to the Chief, presenting a difficult span of control problem. He himself violates the chain of command by giving direct orders to individuals three and four levels below him.

However, instead of the usual lines of authority descending from the top of the chart down, lines were drawn to depict the actual route of reporting within the organization. A solid line was used to indicate a route typically followed by the two connecting boxes. Thus, it showed the way people typically reported upward and also the way that orders typically flowed downward. An example of the Informal Organization Chart can be seen in Figure 3.

In the study we made, we determined that thirteen separate individuals usually reported into the office of the Chief. This meant that he had an excessive Span of Control problem that forced him to come into the office on Saturdays to complete his routine work. Worse than that, the Chief Executive was in the habit of giving orders directly to the individuals that typically came into his office. This meant that he often gave orders to individuals four or five echelons below him without recourse to the chain of command between the two individuals. (In our recommendations we stated in no uncertain terms that the Chief Executive must begin following the chain of command in his orders if he wished his organization to run effectively.) As the reporting lines indicate, an individual would often report to the person two levels above him instead of to his immediate superior. Others would report three levels above. Often, in this organization, an individual would move four levels above him to get an order countermanded that he had received from his immediate superior. Sometimes an individual would not have his subordinates report to him and he, in turn, would not report to his superior. This individual was something of an island unto himself. It was recommended that one of these individuals be located more beneficially elsewhere (almost anywhere, elsewhere). Thus, it can be seen that the Informal Organization Chart indicates most clearly deficiencies in the structure of the organization that can usually be remedied by training courses of the supervisory development type, by ordering all individuals to obey the laid-down chain of command, or by holding conferences to reconcile misunderstandings in the flow of information and determining the optimum route of reporting.

DIFFERING VIEWS

The fifth and last Organization Analysis technique is perhaps the simplest in application and the quickest to achieve beneficial results. This is the technique of having all key individuals in the organization draw the organization chart as they picture it. This technique is extremely useful in companies where no organization chart exists and all individuals are confused because of the lack of a chart. Following the collection of all key people's drawings, a reconciliation is made and the optimum chart decided and decribed to all people concerned. The example I shall use was the result of a study in which Professor Wilson Bentley of Oklahoma A & M University acted as a consultant in a small southwestern concern. Figure 4 depicts the organization as the Vice-President and General Manager of the company visualizes it. Note that the Chief Engineer has six organizational segments reporting directly to him. In Figure 5 we see the organization as the Chief Engineer has drawn it. Note that in Figure 5 there are only four organizations reporting to the Chief Engineer, and one of these, Plant Maintenance, was not even mentioned by the Vice President! One might ask, "How could these two individuals disagree so widely about their jobs?" Note that the Chief Engineer shows the Treasurer as a staff officer with the General Manager sometimes reporting to him. The General Manager indicates the Treasurer as a line official directly

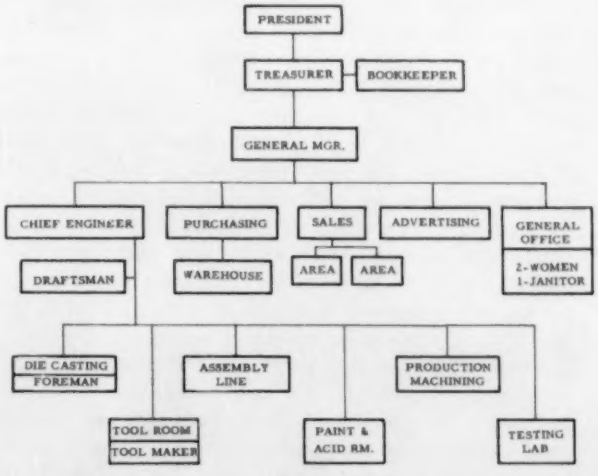

FIG. 4

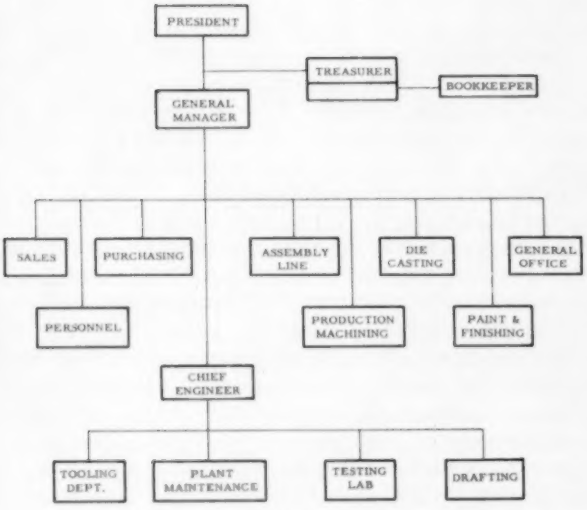

FIG. 5

between himself and the President.

In Figure 6, we see the organization as the Treasurer visualizes it. The Treasurer, a rather modest individual, depicts himself as a staff official reporting usually to the Vice-President and General Manager and sometimes to the President. It might be noted that the Treasurer agrees a little more closely with the Vice President and General Manager in the number of functions reporting to the Chief Engineer. However, he has three of these sometimes reporting to the Vice-President and General Manager.

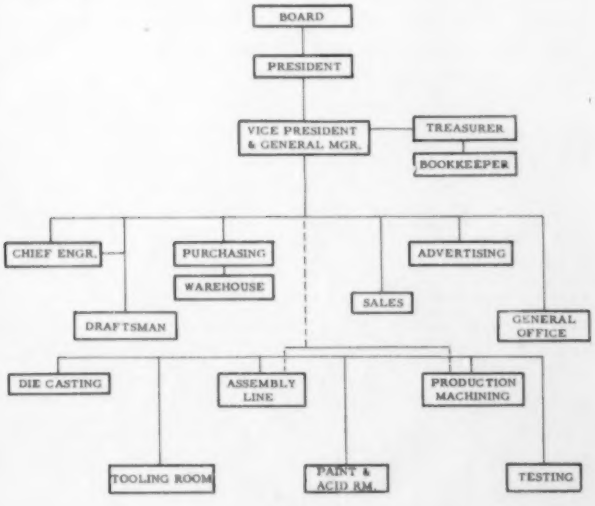

FIG. 6

Following the construction of these three organization charts, the group was gathered together and the optimum organization chart for the company was discussed. The recommended chart appears as Figure 7 and it bears no resemblance to the previous three. It can be seen the Chief Engineer's title, which did not accurately describe his functions, was changed to the Manufacturing Department Supervisor. The assembly line, which the Chief Engineer normally did not think he supervised, was set up as a separate department under an Assembly Department Foreman. Lastly, the Treasurer, an individual operating in a certain amount of confusion, was set up as a separate department called the Treasurer and Comptroller Department with both accounting and the warehouse reporting to him. Thus, the reconciliation of the company's organization problems was achieved and a reasonable amount of satisfaction was given to all.

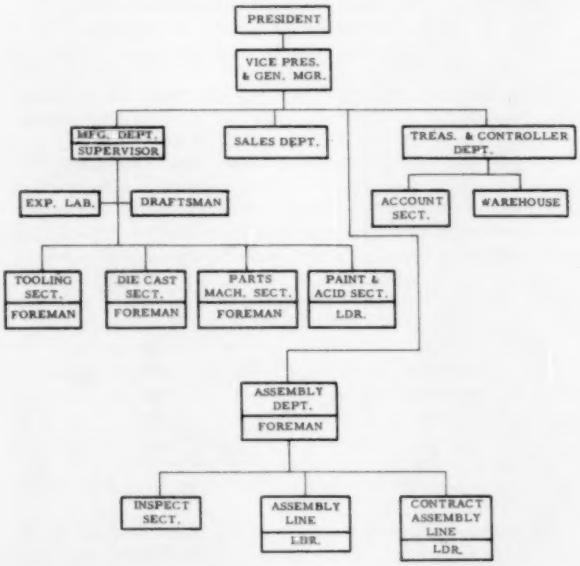

FIG. 7

It is believed that a good Industrial Engineer can apply a Linear Responsibility Chart, Multiple Correlation Chart, a Spectrum Analysis, the Informal Organization Chart, or request the key individuals to draw the organization and then reconcile their opinions to a single solution. These techniques permit an objective approach to the analysis of an organization without forcing the analyst to go into a great amount of detail concerning the work being performed. Only when we can clearly see the organization under analysis can we make the needed improvements to that organization.

The Assignment of Workers in Servicing Automatic Machines

by DOCENT CONNY PALM

Translated from Swedish by Birger Lövgren, Cornell University, Ithaca, N.Y. The original paper occurred in "Industritidningen Norden," Vol. 75 (1947) pp. 75–80, 90–94, 119–123 with the title "Arbetskraftens fördelning vid betjäning av automatmaskiner."

Translators note: In a footnote on page 379 (p. 416, 2nd Edition) in his book "An Introduction to Probability Theory and its Application" (John Wiley and Sons, Inc. 1950, 1st Edition; 1957, 2nd Edition), Professor William Feller refers to the article translated here. Moreover, he gives two examples (pp. 379–383, 1st Edition; pp. 416–421, 2nd Edition) closely related to the subject matter of this article. There are also references to Dr. Palm's article in D. G. Malcolm's article, "Queueing Theory in Organization Design," in JOURNAL OF INDUSTRIAL ENGINEERING, *Vol. 6, November–December 1955, pp. 19–26 and in R. N. Fetter's article "The Assignment of Operators to Service Automatic Machines," in the same* JOURNAL, *Vol. 6, September–October 1955, pp. 22–30. Frequent reference to Palm's work has resulted in the article being quite well known to statisticians and Industrial Engineers in this country. Because it is written in Swedish, however, very few of those who might be interested in it have been able to read it. Therefore, Publication of the work in English seems desirable. The original article consisted of three parts but the translation has been given in one. With the exception of a few omissions to avoid unnecessary repetitions, the translation is intact. Everything important is included; all mathematical reasonings and derivations, all examples, and all tables and graphs.*

The translator is greatly indebted to Mr. John E. Romaine for reading the early drafts, and for putting all the commas where they should be. The translator also wishes to thank Professor John H. K. Kao, Dept. of Industrial and Engineering Administration, Cornell University, for many helpful suggestions.

ARTICLE NO. 1

THIS article gives the results of an investigation of the waste times which arise when a group of automatic machines are serviced by one and the same worker. A method for calculating the economical optimal number of machines per worker is given. This method has been successfully used in the contract calculations at Malmö Yllefabriks *AB* and Manufaktur *AB* Malmö. (Spinning-mills and textile-industries.)

In many industries several automatic machines are serviced by one and the same worker. The necessary work required for each machine, consisting of feeding in new material and making possible adjustments or corrections, is seldom enough to occupy all the time of the worker. It is then possible, without unnecessary stops, to let the same worker do some or all of the necessary operations on several automatic machines. That such an arrangement pays, depends mostly on the fact that occasions for intervention, by the worker, on each machine usually occur with rather long intervals between them. If the calls for service by each machine are too frequent, and the adjustments required can be done very quickly, one might fear that the transit time for the worker, between the machines in need of service, will have an unfavorable influence on the possible amount of "useful" work.

If it seems suitable to let each worker service several machines, the question arises as to what number of machines per worker is the most profitable. At first sight the problem seems to be simple. One lets the total servicing time for the machines be equal to the effective working time, which we can expect from the worker, with regard to such things as his transit times between the different machines. Because of the economical consequences, one is, however, not allowed to solve it in so simple a way. It can happen that one machine requires service while the worker is busy with another machine. The first machine will then have to wait, which results in an unnecessary cost. By reducing the number of machines per worker one can diminish these waiting times. It is, however, uncertain that such a reduction will pay, since the cost of the worker per machine will consequently increase. It is evident that there will be an economical optimum at which the cost for the waiting times of the machines will balance the advantage of having one worker serve several machines. It is further evident, that this optimum depends both on the ratio between the cost of the waiting times for the machines and the cost for the worker, and also on the necessary service time for each machine.

It is important, economically, to be able to calculate this optimum number of machines per worker. This problem seems to have been noticed during recent years and several solutions have been proposed. In these, the mathematical methods used are based upon the assumption that the servicing times for the machines occur independently of each other, and so the times when the worker is needed are randomly distributed. Then one can base the mathematical treatment on the theory of probability. The solutions proposed hitherto do not seem to be consistent in the mathematical treatment nor to have been trusted by professional people (3).

In the following, a new treatment of the problem is given, which seems to be mathematically correct, and which makes the numerical work rather simple. The results have, therefore, been presented in a diagram which will probably suffice for most practical purposes. Here we might mention that the mathematical treatment of the problem is very similar to certain waiting time problems occurring in telephony. For the problems of telephony, which are usually much more complicated than the

problem treated here, good mathematical methods of solution have long existed. The derivations given below are based on these methods.

We now define the following characteristic quantities, which are inherent in the working schedule of an automatic machine.

Machine Time: The average of the total time per unit time when the automatic machine is working.

Servicing Time: The average of the total time per unit time when the worker adjusts, empties, or reloads the machine.

Idling Time: The average of the total time per unit time when the automatic machine is not working.

The sum of these three times must, of course, always be one. In this connection we don't care about the idling times which are caused by things other than worker inability to service more than one machine at a time.

As an example we can take an automatic machine which, during an 8 hour shift, works for 6.5 hours, is serviced for one hour and idles for one half hour. The machine time will then be, in per cent, 81.25, the servicing time 12.5, and the idling time 6.25.

As an initial quantity for the computation, we use the quotient between the servicing time and the machine time, which we can always calculate knowing data on the machine and the kind of work to be done. This quotient, which we will call k, is of course, independent of the idling time.

If we now call the machine time m, the servicing time b, and the idling time d, we get:

$$m + b + d = 1 \qquad \text{Eq. 1.}$$

From the definition of k:

$$k = \frac{b}{m} \qquad \text{Eq. 2.}$$

These two relations, in which k is known, are not sufficient to determine the three unknown quantities m, b, and d. In order to do so we need one more relation, and to get this relation we must know the number of automatic machines per worker. To get a result which has practical value, we must assume that every worker only operates machines with the same k value. This assumption can be considered rather natural and does not imply any restriction on the usefulness of the results. Besides, it is rather rare that plant superintendents make this computation every time the k value of an automatic machine is changed. It is sufficient to make these calculations once and for all based on the average of the k values which the different machines and the different materials have.

There is, however, another way which can be of interest in practice, to generalize the problem. If we have several groups of automatic machines, each group operated by one man, we will always get, at least theoretically, a better use of the available labor and therefore a reduction in the idling time, by placing all the machines in one group which are operated by several workers. Instead of reduced idling time, we can get a reduction in the necessary number of workers. However, in practice such an arrangement will often cause difficulties in organization, which can make the gain doubtful. With this exception the gain can easily be calculated, and at the end of this article there is an example.

In the beginning, we will assume that every machine is serviced by only one worker. The number of machines per worker is n, and all the machines belonging to the same worker have the same k value. From this k value we will first calculate the idling time per machine. From the idling time we can easily calculate several other quantities, for example, the performance index of the worker and the machines. The idling time is, however, the quantity which is the most useful for the calculation of the n value which gives an economical optimum. This part of the calculations is very simple and will be explained later.

DERIVATION OF THE BASIC FORMULAS

In order to calculate the idling time we must first define several quantities which characterize the performance of the group of automatic machines.

P: The average of the total time per unit time when all the n machines are working.

T_0: The average of the total time per unit time when one of the machines is serviced by the worker and all the other $(n-1)$ machines are working.

T_q: The average of the total time per unit time when one of the machines is serviced by the worker, q of the other machines are idle and waiting for the worker, and the other $(n-1-q)$ machines are working.

In the last case, q can have any of the values 1, 2, and up to $(n-1)$ so we have $(n-1)$ different quantities, T_q.

The quantities P and T_q characterize completely the performance of the group of the automatic machines; that is, at every moment, some of these states must prevail. The sum of all of these quantities must give the total time, which, per unit time, is one. That is:

$$P + \sum_{q=0}^{n-1} T_q = 1 \qquad \text{Eq. 3.}$$

Now we will show how to find a sufficient number of relations between these quantities in order that they can be uniquely determined. We will do this with the help of the theory of probability.

The starting point for these calculations is the assumption that the occasions when the worker has to intervene, are randomly distributed. Even a shallow examination of the way automatic machines work gives a strong motivation for this assumption. In addition, all earlier attempts to solve our problem have been based on the same assumption.

If there is a need for intervention by the worker when one of the machines stops, we say that we have a call for service. According to the assumption, we will have these calls randomly distributed over all the working time. This means that for every machine working, the probability that it will give a call for service during the time t will always have the same magnitude and be independent

of the time already worked. On the other hand, this probability is, of course, dependent on the interval t. In the mathematical treatment it is most convenient to consider the probability that there will be no call during the time interval t. This probability is $\phi(t)$.

We can derive the expression $\phi(t)$ by the following reasoning. We start with an arbitrary time interval and consider the probability that no machine will call for service during the following time interval (t_1+t_2). This probability is, by definition, $\phi(t_1+t_2)$. However, this probability is also equal to the product of the probability that no call will occur during the first part t_1 of the interval, that is, $\phi(t_1)$, and the probability that no call will occur during the part t_2 of the interval, that is $\phi(t_2)$. That we get the probability of the occurrences of both these cases by multiplying the probability of each case, depends on the fact that both of the probabilities according to the basic assumption of random distribution of the calls, are quite independent of each other. Thus we have:

$$\phi(t_1 + t_2) = \phi(t_1) \cdot \phi(t_2) \qquad \text{Eq. 4.}$$

This relation will now be valid for arbitrary values of t_1 and t_2. One can now show that this functional relationship is satisfied only if $\phi(t)$ has an exponential form which can be written:

$$\phi(t) = e^{-yt} \qquad \text{Eq. 5.}$$

Here y is a constant, the meaning of which will be explained later. It is easy to see that this function satisfies Eq. 4.

The exponential form e^{-yt} of the distribution function[1] for random occurrences has long been known and is used in all investigations of random probability processes. It can be derived in many ways; the derivation above is given because of its shortness.

The distribution function $\phi(t)$ according to Eq. 2. has, for $t=0$, the value 1, and decreases toward zero for increasing t values, which is what we might expect from its definition as a probability. In order to show the meaning of the constant y, we are now going to calculate the probability that the working time for one machine is exactly t units of time, that is, that it has a length between t and $t+dt$, where dt is an infinitesimal quantity. If this occurs, the following two mutually independent events must occur:

1. During the time t there is no call for service from the machine. The probability for this is e^{-yt}.
2. During the interval dt there is a call from the machine. The probability that there is a call is then 1 minus this expression. that is, $1-e^{-ydt}$.

The probability that both these cases will occur is now the product of the probabilities for each case, that is:

$$e^{-yt}(1 - e^{-ydt})$$

From the series expansion for the exponential function we obtain:

$$1 - e^{-ydt} = ydt - \tfrac{1}{2}(ydt)^2 + \cdots$$

Because dt is a differential quantity, we can neglect all terms with higher exponents in dt. We then obtain that:

$$ye^{-yt}dt \qquad \text{Eq. 6.}$$

is the probability that the time from the starting of the machine until the moment when it needs adjustment is exactly t. Now we can calculate the average of this time. We obtain this average by multiplying t by the probability in Eq. 6. and then adding the products for all possible values of t. Because the probability is a continuous function this sum is the following integral:

$$\int_0^\infty yte^{-yt}dt$$

The value of this integral is $1/y$. That is, y is the reciprocal of the average of the individual working times of the machine. This constant y is supposed to be the same for all machines in the group under consideration. This agrees with the assumption that k has the same value for all machines in the group.

The distribution function in Eq. 5. is not sufficient to obtain the necessary relationships between all the introduced quantities. We must also know the distribution function for the servicing times. However, we can not derive directly the necessary mathematical expression for this function, for example one of the same type as Eq. 5. In Eq. 5. we got the exponential form as the necessary consequence of the assumption of the random distribution of the calls for service. We have no analogy to this when considering the servicing times. However, it seems to be natural, even in this case, to start with a distribution function of exponential form. A very strong reason for this is that the mathematical treatment will be very much facilitated. Besides, we need not fear that the form of this rather special distribution function will restrict the practical applications of the results. From the results of extensive investigations, using very advanced mathematical methods in telephony (2), we know that the form of the density function has only a very slight effect on the numerical results. Besides, from experience, we know that the servicing times vary considerably. It is therefore not true to assume that all the servicing times have the same length. The exponential distribution, however, gives a good statistical description of the rather randomly varying servicing times. As it is impossible to measure and take into account the different distribution functions in the calculations for all different possibilities of working schedule, we will always have to choose some special form of distribution function. It seems natural to choose the form that gives the simplest mathematical treatment.

[1] According to convention, $1-e^{-yt}$ rather than e^{-yt} is a distribution function. However, since the author uses the name "distribution function" for e^{-yt} the translation will follow the author's notation.

The distribution function will then be:

$$e^{-t/s} \qquad \text{Eq. 7.}$$

This expression gives the probability that one servicing time lasts at least the time t. s is a constant. As before we can show that s is the average of the servicing times. We now assume that s is the same for all machines in the group for the same reason that we assume this for y.

We now find that:

$$\frac{1}{s} e^{-t/s} dt \qquad \text{Eq. 8.}$$

gives the probability that one servicing time will last exactly the time t. Another property of the distribution function Eq. 7. has a special importance in the following: Assume that one servicing time has already lasted for the time x. We then inquire about the probability that it will last t more units of time. We will get that probability by dividing the probability for a servicing time of at least $x+t$ by the probability for a servicing time of at least the time x, that is:

$$\frac{e^{-(x+t)/s}}{e^{-x/s}} \qquad \text{Eq. 9.}$$

This, however, is $e^{-t/s}$ and is independent of x. That is, the continued length of a servicing time is independent of the time it has already lasted. This means that when considering a state in the group of machines we need not know how long the state has already existed. This causes simplification of the mathematical treatment if one assumes the exponential form of Eq. 7.

We can now write down the relations between the quantities P and T_q. We consider first the state in the group of machines when all the machines are working. We call this state P. Now we calculate the probability that such a state will last the time t before it will be interrupted, which happens as soon as any of the machines gives a call. For each of the machines the probability that a machine will not give a call during at least the time t, is e^{-yt}, and this is from an arbitrarily chosen time. As the calls from the different machines, according to our assumptions, are independent of each other, the probability that none of the machines will give a call during the time t is equal to the product of the probabilities for each of the machines; that is,

$$e^{-nyt} \qquad \text{Eq. 10.}$$

This is the distribution function for the state P. In the same manner as before, we find that the average time that this state will last is now $1/ny$. Now, P is the average of the total time per unit time when all the machines are working. It is evident that this time P, divided by $1/ny$, gives the average number of occurrences of state P per unit time. This number is:

$$ny\,P \qquad \text{Eq. 11.}$$

We will now consider the state T_q, which occurs when one of the machines is serviced by the worker and q of the machines are waiting for service, while the remaining $n-1-q$ machines are working. We now inquire into the probability that this state will last at least the time t. It can be interrupted:

1. By a call for service from one of the working machines. If this happens, we will get the state T_{q+1}.
2. By the worker being ready with the machine he is servicing. If this is the case, we will then get the state T_{q-1}.

The probability that none of the working $n-1-q$ machines will stop during the time t is now the product of the probabilities for each of the machines not giving a call during this time. That is:

$$e^{-(n-1-q)yt} \qquad \text{Eq. 12.}$$

The probability that the worker during the time t will not be ready with the machine he is servicing is, according to Eq. 7.:

$$e^{-t/s} \qquad \text{Eq. 13.}$$

The probability for neither case 1 nor case 2 occurring is the product of the probabilities in Eq. 12. and Eq. 13., that is:

$$e^{-(n-1-q)yt-t/s} \qquad \text{Eq. 14.}$$

If we now calculate by integration, the average value, we find

$$\frac{1}{(n-1-q)y + \frac{1}{s}} \qquad \text{Eq. 15.}$$

which is the average length of a state when q machines are waiting, one is being serviced, and all the other machines are working. Now T_q was the average duration of all such states per unit time. Then T_q divided by Eq. 15. gives the average number of times per unit time when this state will occur:

$$\left\{(n-1-q)y + \frac{1}{s}\right\} T_q \qquad \text{Eq. 16.}$$

It is easy to see that this result is valid even if q is equal to 0; that is, when one machine is serviced but no machines are waiting. If the worker is ready before a new call for service occurs, this state T_0 will change in P. The expression Eq. 16. is also valid for the other case when $q=n-1$; that is, when one machine is serviced and all the others are waiting.

The state T_q can disappear in two different ways: by a call from one of the working machines or by the worker being finished with the machine he was servicing. We can now find the probability for the occurrence of each of these cases. First we consider the last case. Assume that this happens at the time t after the beginning of the state. None of the $n-1-q$ working machines should call during this time and the probability for this is:

$$e^{-(n-1-q)yt}$$

The probability that the worker is ready at exactly the time t is expressed by Eq. 8. The probability for both cases will then be the product of the probabilities for each case and we get:

$$\frac{1}{s} e^{-(n-1-q)yt-t/s}dt$$

For every value of t the worker can be ready before any call occurs and we get the total probability for this by integrating the expression above between $t=0$ and $t=\infty$. Thus we get:

$$\frac{1}{s}\int_0^\infty e^{-[(n-1-q)y+1/s]t}dt$$

which is:

$$\frac{\frac{1}{s}}{(n-1-q)y+\frac{1}{s}} \qquad \text{Eq. 17.}$$

The probability for the opposite case, that is, that a call will occur before the worker is ready with his machine must then be 1 minus the expression in Eq. 17:

$$\frac{(n-1-q)y}{(n-1-q)y+\frac{1}{s}} \qquad \text{Eq. 18.}$$

By multiplying the average number of times per unit time when the state T_q occurs (Eq. 16.) by the probability that it will be finished before any new call occurs, (Eq. 17.), we get the average number of times per unit time when a state T_q is finished by the worker being ready before any new call occurs. This will be:

$$\frac{1}{s} T_q \qquad \text{Eq. 19.}$$

As we can see, this is valid even for $q=0$.

In the same way we get, by multiplying Eq. 16. by Eq. 18., the average number of times per unit time when the state T_q is finished by a new call from one of the machines before the worker is ready. This is:

$$(n-1-q)yT_q \qquad \text{Eq. 20.}$$

Thus we have gotten an expression for the average number of times per unit time when the different states occur and when they change into each other in the group of machines under consideration. It is now possible to obtain the desired relations. These are based on the fact that the number of times which a state of a certain kind occurs must equal the number of times when it arises from other states. First we consider the state T_q. From Eq. 16. we get the average number of times per unit time when the state occurs. Now this state can arise in two ways, that is:

1. From the state T_{q+1} by the worker being finished with a machine and going over to one of the waiting machines.
2. From the state T_{q-1} by a call for service occurring from one of the working machines.

We get the average number of times per unit time when 1. occurs from Eq. 19.:

$$\frac{1}{s} T_{q+1}$$

We get the average number of times per unit time when 2. occurs from Eq. 20.:

$$(n-q)yT_{q-1}$$

Now the expression in Eq. 16. must equal the sum of both of these expressions and we get the relation:

$$\left\{(n-1-q)y+\frac{1}{s}\right\} T_q = \frac{1}{s} T_{q+1} + (n-q)yT_{q-1} \qquad \text{Eq. 21.}$$

Here we can remark that we have assumed that the state T_q can only arise from the states T_{q-1} and T_{q+1}. Now perhaps one can ask whether the state T_q can arise from the state T_{q-2}, for example, by calls from two of the working machines. If, however, these calls are not exactly simultaneous we will have the state T_{q-1} as an intermediate state, which means that it has already been taken care of in Eq. 21. Besides, the probability that two calls will occur simultaneously seems to consist of the product of the two differentials. This means that the average number of times per unit time, when such a thing will occur, is zero, mathematically speaking. From this we can see that we need only consider the two states T_{q-1} and T_{q+1}.

When writing down Eq. 21. we assumed that the state T_q could arise from the state T_{q+1} as well as from the state T_{q-1}. This is, of course, possible only if q is not $n-1$ or 0. If $q=n-1$, the state T_q can only arise from the state T_{q-1} by a call from the only working machine. Then we get the specific relation:

$$\frac{1}{s} T_{n-1} = yT_{n-2} \qquad \text{Eq. 22.}$$

If $q=0$, the state T_0 can arise from the state T_1 by the worker being finished with a machine or from the state P by a call from one of the working machines. Then we get the relation:

$$\left\{(n-1)y+\frac{1}{s}\right\} T_0 = \frac{1}{s} T_1 + nyP \qquad \text{Eq. 23.}$$

At last we can obtain a specific relation even for the state P. This state can disappear only by a call for service from one of the n working machines and it can only arise from the state T_0 by the worker being finished with a machine. Evidently, we get the relation:

$$nyP = \frac{1}{s} T_0 \qquad \text{Eq. 24.}$$

The relations Eq. 21. to Eq. 24. give a system of $n+1$ linear equations for the $n+1$ unknown quantities T_q and P. The system is homogeneous, however, and so we need one more relation, which we have in Eq. 3. One can show that there is only one unique solution of these equations. In order to obtain this solution we can first simplify the equations somewhat. If we add Eq. 24. to Eq. 23. we obtain:

$$(n-1)yT_0 = \frac{1}{s}T_1$$

If we then add this relation to the relation we get from Eq. 21. for $q=1$, we get:

$$(n-2)yT_1 = \frac{1}{s}T_2$$

By going on in the same way, we obtain in general:

$$(n-q)yT_{q-1} = \frac{1}{s}T_q \qquad \text{Eq. 25.}$$

By using this equation, we can express all the T_q in P. We obtain:

$$T_q = (n-q)(n-q+1)\cdots n(sy)^{q+1}P \qquad \text{Eq. 26.}$$

From Eq. 3. we get:

$$P\left\{1 + \sum_{q=0}^{n-1} \frac{n!}{(n-q-1)!}(sy)^{q+1}\right\} = 1 \qquad \text{Eq. 27.}$$

from which P can be evaluated.

By Eq. 26. and Eq. 27. all the quantities P and T_q are determined. The simplest way to make the numerical calculations is to use Eq. 25. recursively. Here we note that the product, sy, but never s and y separately, occurs in these formulas.

The idling time for each machine is d. Because all the machines are assumed to have the same average idling time, the idling time for the n machines in the group under consideration is nd. For each time in which we have a state T_q, we get a contribution to this total idling time, and this contribution is evidently qT_q, because q machines are waiting during the state T_q. Thus we get the relation:

$$nd = \sum_{q=1}^{n-1} qT_q \qquad \text{Eq. 28.}$$

from which the idling time per machine, d, can be calculated by knowing the quantities T_q. Furthermore, we can calculate the servicing time. The total servicing time for all machines is nb and this equals the sum of all the states when the worker is servicing one machine, that is:

$$nb = \sum_{q=0}^{n-1} T_q$$

which by relation Eq. 3. can be written:

$$nb = 1 - P \qquad \text{Eq. 29.}$$

Finally from relation Eq. 1. we can calculate the machine time (m).

The expression in Eq. 28. can be written in a form which makes the numerical calculations simpler. We write Eq. 25. in the form:

$$T_{q+1} = sy(n-1)T_q - syqT_q$$

If we add all the relations which we get from this equation for $q=0, 1, \cdots n-1$, we get:

$$\sum_{q=0}^{n-2} T_{q+1} = sy(n-1)\sum_{q=0}^{n-2} T_q - sy\sum_{q=1}^{n-2} qT_q$$

and after some calculation:

$$sy\sum_{q=1}^{n-1} qT_q = \{sy(n-1)-1\}\sum_{q=0}^{n-1} T_q + T_0$$

Applying Eq. 3. and Eq. 24. we get this:

$$\sum_{q=1}^{n-1} qT_q = n - 1 + P - \frac{1-P}{sy} \qquad \text{Eq. 30.}$$

The sum in Eq. 28. can thus be evaluated from the quantity P. Then we get the expression for the idling time d:

$$d = 1 - \left(1 + \frac{1}{sy}\right)\frac{1-P}{n} \qquad \text{Eq. 31.}$$

From this formula and from Eq. 1. and Eq. 29. we get the machine time:

$$m = \frac{1-P}{nsy} \qquad \text{Eq. 32.}$$

From Eq. 29. finally we get:

$$k = \frac{b}{m} = sy$$

Thus we have found that the product of y and s equals the constant k, which characterizes the working of the machines and which must be given before numerical calculations can be started.

ARTICLE NO. 2

METHODS OF NUMERICAL TREATMENT

We will now make an abstract of the final formulas and the method of calculation. With the formulas:

$$T_0 = nkP \qquad \text{Eq. 33.}$$

and

$$T_q = (n-q)kT_{q-1} \qquad \text{Eq. 34.}$$

we get all the quantities T_q expressed in P. Here, q goes up to $n-1$. After this we calculate P by the formula:

$$P + \sum_{q=0}^{n-1} T_q = 1 \qquad \text{Eq. 35.}$$

P means the total time per unit time when all the machines are working. Then $1-P$ is the time when the worker is servicing the machines and can be called the worker performance index. The servicing time for every machine we obtain from:

$$b = \frac{1 - P}{n} \qquad \text{Eq. 36.}$$

The machine time for each machine we obtain from:

$$m = \frac{b}{k} \qquad \text{Eq. 37.}$$

And finally we get the idling time for each machine from:

$$d = 1 - m - b \qquad \text{Eq. 38.}$$

One more quantity of interest is the machine performance index which can be defined as $b+m$. This is the part of the time when a machine is working or is being serviced.

In order to show how simple the calculations are we will give a few examples:

Example 1. Assume that $n=6$ and $k=0.1$, that is, 6 machines and a quotient between the attending times and the machine time of 0.1. By using Eq. 33. and Eq. 34. we can arrange the calculations in the following way:

P	1.00000
$T_0=6\times0.1\ P$	0.60000
$T_1=5\times0.1\ T_0$	0.30000
$T_2=4\times0.1\ T_1$	0.12000
$T_3=3\times0.1\ T_2$	0.03600
$T_4=2\times0.1\ T_3$	0.00720
$T_5=1\times0.1\ T_4$	0.00072
Total	2.06392

P is now 1 divided by this sum. We obtain $P=0.4845$. From this we get the worker performance index, $1-P=0.5155$. The servicing time b is obtained by dividing this by 6 and we get $b=0.0859$. The machine time m is obtained by dividing b by k, and we get $m=0.8592$. Finally we get the idling time $d=1-b-m=0.0549$. The machine performance index is 0.9451.

Example 2. Assume that $n=20$ and $k=0.03$.

We obtain a table as in the previous example:

P	1.00000
$T_0=20\times0.03\ P$	0.60000
$T_1=19\times0.03\ T_0$	0.34200
$T_2=18\times0.03\ T_1$	0.18468
$T_3=17\times0.03\ T_2$	0.09419
$T_4=16\times0.03\ T_3$	0.04521
$T_5=15\times0.03\ T_4$	0.02034
$T_6=14\times0.03\ T_5$	0.00854
$T_7=13\times0.03\ T_6$	0.00333
$T_8=12\times0.03\ T_7$	0.00120
$T_9=11\times0.03\ T_8$	0.00040
$T_{10}=10\times0.03\ T_9$	0.00012
$T_{11}=9\times0.03\ T_{10}$	0.00003
$T_{12}=8\times0.03\ T_{11}$	0.00001
Total	2.30005

We get $P=0.4348$. The worker performance index, $1-P=0.5652$. Servicing time $b=0.0283$. The machine time $m=0.9420$. The idling time $d=0.0297$. The machine performance index $=0,9703$.

Of course one need not always use such high accuracy as in these examples. For example, if we had chosen to use only three decimals in example 2, we would only have had to calculate the quantities T_0 to T_8 and could have neglected T_9 to T_{19}. These two examples are rather interesting because they show the great influence of the number of machines. In both examples, the product nk has the same value and equals 0.6. If we had no idling time we would have gotten the same worker performance index. However, in the first case we had 5.5% idling time but in the second case only 3.0%.

The mathematical assumptions which form the basis for the resulting formulas, Eq. 33. through Eq. 35. are: the random distribution of the calls for service, and the exponential form, Eq. 7., of the distribution function for the servicing times. The first condition can be considered to be satisfied with sufficient accuracy so that the usefulness of the results is not impaired. This, of course, is invalid if one takes measures to distribute the calls in some special way. Such measures are probably not possible to carry through anyway, and so we need not worry for the validity of the formulas in this respect. Regarding the assumption of the exponential form for the distribution function, we can refer to the experience we have obtained from research done in the field of telephony in the study of waiting problems where many problems are similar to those considered here. It is true that the results depend on the form of the distribution function, but this dependence is slight and when the servicing times vary somewhat, we need not worry about the difference between the real and the assumed form of the distribution function. Here we can note that the methods used earlier for the idling-time problem do not restrict the distribution function for the servicing times to any special form. This, however, is hardly to be regarded as an advantage, because it is due to the fact that these methods are based on an insufficient or incorrect mathematical analysis of the problem.

As a conclusion we might state that there are strong reasons for believing that the method outlined above will give a very good description of the real circumstances. In the application one must, of course, use the formulas in the right way. Here, I would like to make a remark. We must, of course, make measurements of the actual servicing times to get the value of k. In order to be able to apply the formulas we must, then, in the servicing-times, take into account all the movements of the worker in the servicing of a machine and this independent of how we usually (e.g. regarding contract calculations) classify his movements. This is mentioned here especially because of the transit-times, that is, the time it takes for

the worker to move from one machine to another. These times must be included in the servicing times. This will complicate, at least in principle, the determination of the optimal number of machines per worker. In a careful analysis we will also have to consider that the transit-time will increase with increasing number of machines, and this means that the quantity k is dependent on the number of machines per worker. In practice, however, this will have a rather small influence, because one can first find a rough value of n and from that value, calculate a value of the transit-time. If one is careful, one can always use a method of successive approximations, by calculating a rough value of n, correcting the k value used, and then calculating a better n value.

In order to be able to use the results in Eq. 33. through Eq. 38. to calculate the optimal number of machines per worker, we must have curves of the idling-times as functions of k and n. We can facilitate this by a special method. To describe this method we define:

$$L_0 = 1$$

$$L_1 = \frac{L_0}{k}$$

and in general:

$$L_r = \frac{L_{r-1}}{r \cdot k} \qquad \text{Eq. 39.}$$

For a group of n machines we then have:

$$P = \frac{L_n}{L_0 + L_1 + \cdots + L_n} \qquad \text{Eq. 40.}$$

and

$$T_q = P \frac{L_{n-q-1}}{L_n} \qquad \text{Eq. 41.}$$

To prove this, it is sufficient to show that the recursion formulas, Eq. 33. and Eq. 34., and the condition Eq. 35. are satisfied. This is easy to show.

Now we calculate a suitable number of L quantities for each k value. By Eq. 40., we can then calculate P and, therefore, the idling-time for all values of n. The advantage of using this method is that we need not calculate a lot of T quantities for each combination of k and n, as in the examples above. We calculate only a row of such quantities for each k. By this method one would have to do only a small part of the calculation work, otherwise necessary, to get sufficiently good tables.[2]

Table 1 gives an idea of the calculations. It is valid for $k = 1$.

It is quite unnecessary to go further than to $n = 15$, in this case, because it will never be profitable to have idling-times of more than 30%. The calculations above have assumed that there is only one man attending each group of machines. As mentioned earlier, one can always obtain a better usage of the workers (at least, theoretically) if several workers cooperate in servicing a larger group of machines. We will give the formulas for this case without the derivations, which, in principle, are similar to those outlined above.

TABLE 1

n	L_n	ΣL_n	P	b	w	d
1	10.000	11.000	0.90909	0.0909	0.9091	0.0000
2	50.000	61.000	0.81967	0.0902	0.9016	0.0082
3	166.667	227.667	0.73206	0.0893	0.8931	0.0176
4	416.667	644.334	0.64666	0.0883	0.8833	0.0284
5	833.333	1 477.667	0.56395	0.0872	0.8721	0.0407
6	1 388.89	2 866.56	0.48451	0.0859	0.8592	0.0549
7	1 984.13	4 850.68	0.40904	0.0844	0.8442	0.0714
8	2 480.16	7 330.84	0.33832	0.0827	0.8271	0.0902
9	2 755.73	10 086.57	0.27321	0.0808	0.8075	0.1117
10	2 755.73	12 842.31	0.21458	0.0785	0.7854	0.1361
11	2 505.21	15 347.52	0.16323	0.0761	0.7607	0.1632
12	2 087.67	17 435.19	0.11974	0.0734	0.7336	0.1930
13	1 605.90	19 041.09	0.08434	0.0704	0.7044	0.2252
14	1 147.07	20 188.16	0.05782	0.0674	0.6737	0.2589
15	764.72	20 952.88	0.03650	0.0642	0.6423	0.2935

The number of machines in the group is n and these machines are attended by p workers. When the p workers are not all busy servicing machines, no machine need wait for attention. We define:

P_μ: The average of the total time per unit time, when μ workers are attending μ machines and the $n-\mu$ remaining machines are working. μ can have the values 0, 1, $\cdots$, $p-1$.

T_v: The average of the total time per unit time when all p workers are attending p machines and v remaining machines are waiting for attention while the $n-p-v$ remaining machines are working. v can have the values 0, 1, $\cdots$, $n-p$.

We obtain the relations.

$$\left.\begin{aligned} P_1 &= nkP_0 \\ 2P_2 &= (n-1)kP_1 \\ 3P_3 &= (n-2)kP_2 \\ &\cdots\cdots\cdots\cdots \\ (p-1)P_{p-1} &= (n-p+2)P_{p-2} \end{aligned}\right\} \qquad \text{Eq. 42.}$$

and

$$\left.\begin{aligned} pT_0 &= (n-p+1)kP_{p-1} \\ pT_1 &= (n-p)kT_0 \\ pT_2 &= (n-p-1)kT_1 \\ &\cdots\cdots\cdots\cdots \\ pT_{n-p} &= 1 \cdot k \cdot T_{n-p-1} \end{aligned}\right\} \qquad \text{Eq. 43.}$$

and finally the non-homogeneous relation

$$\sum_{\mu=0}^{p-1} P_\mu + \sum_{v=0}^{n-p} T_v = 1 \qquad \text{Eq. 44.}$$

[2] One can show that the Eq. 40. equals E_1, $n(1/k)$. Here E_1, $n(x)$ is a function which is much used in telephony, and extensive tables will soon be published. See also reference (1).

These relations determine the quantities P_μ and T_ν uniquely. The idling-time d per machine can then be calculated from

$$d = \frac{p}{nk} T_0 + \left(1 - \frac{p}{n} \cdot \frac{1+k}{k}\right) \cdot \left(1 - \sum_{\mu=0}^{p-1} P_\mu\right) \quad \text{Eq. 45.}$$

and the machine-time and the servicing-time from:

$$\left.\begin{aligned} m &= \frac{1-d}{1+k} \\ b &= km \end{aligned}\right\} \quad \text{Eq. 46.}$$

The numerical calculations are no more complicated than when $p=1$. As an example we will consider the following:

Example 3. Assume $n=20$, $p=3$, $k=0.1$, that is, 20 machines which are attended by 3 workers and the quotient between servicing and machine-time equal to 0.1.

We arrange the calculation in the same way as in Examples 1 and 2 and obtain:

P_0	$=1.000\ 00$	$0.136\ 25$
$P_1 = 20\times0.1\ P_0$	$=2.000\ 00$	$0.272\ 50$
$P_2 = 19\times0.1\ P_1$	$=1.900\ 00$	$0.258\ 88$
$T_0 = 18\times0.1\ P_2$	$=1.140\ 00$	$0.155\ 32$
$T_1 = 17\times0.1\ T_0$	$=0.646\ 00$	
$T_2 = 16\times0.1\ T_1$	$=0.344\ 53$	
$T_3 = 15\times0.1\ T_2$	$=0.172\ 27$	
$T_4 = 14\times0.1\ T_3$	$=0.080\ 39$	
$T_5 = 13\times0.1\ T_4$	$=0.034\ 84$	
$T_6 = 12\times0.1\ T_5$	$=0.013\ 93$	
$T_7 = 11\times0.1\ T_6$	$=0.005\ 11$	
$T_8 = 10\times0.1\ T_7$	$=0.001\ 70$	
$T_9 = 9\times0.1\ T_8$	$=0.000\ 51$	
$T_{10} = 8\times0.1\ T_9$	$=0.000\ 14$	
$T_{11} = 7\times0.1\ T_{10}$	$=0.000\ 03$	
$T_{12} = 6\times0.1\ T_{11}$	$=0.000\ 01$	
	Total $7.339\ 46$	

It is not necessary to calculate the quantities T_{13}–T_{17} when we are using 5 decimal places. The P and T quantities are now obtained by dividing the numbers in the next to the last column by the sum 7.33946. Then we obtain the values at the beginning of the last column. From this we obtain:

$$P_0 + P_1 + P_2 = 0.66763$$

From Eq. 45. and Eq. 46. we obtain finally:

$$d = 0.01694 \qquad m = 0.89369$$
$$b = 0.08937$$

Comparing this result with the values in Table 1, which are valid for the same k value but with $p=1$, we see that approximately the same idling-time is obtained as with three machines per worker. By arranging for cooperation among three workers, the number of machines per worker increases from 3 to 6.7 without changing the idling-time. This must be considered as a very remarkable result which should give rise to an investigation of the possibilities of arranging for cooperation between machine operators to a larger extent than is now the case.

DETERMINATION OF ECONOMICAL OPTIMUM

Now we will show how to compute the economically optimal number of machines per worker from the relation between the idling-time and the number of machines. Here it is necessary to go into a simple discussion regarding methods of calculating costs, and we are not going to examine these methods any more closely than this.

The quantity which we are going to minimize is the unit cost of the manufactured product. To get rid of a lot of unnecessary constants we choose the quantity manufactured per unit time by one working machine as our unit. (Here we assume that the machines are working continuously without any interruption.) This cost is then Q_0 and is made up of the cost of the machine, the cost of the material used in the manufacturing, and the wages of the workers.

We define:

Q_m: Cost per unit time for a working machine.
Q_b: Cost per unit time for a machine being serviced.
Q_d: Cost per unit time for an idling machine.

These costs can be determined quite independently of k, and the number of machines per worker. In Q_m we must include the cost of the material fed into the machine and the power cost. The costs for depreciating the machine, for the machine locality, and other such things, shall usually be included as equal parts in Q_m, Q_b and Q_d. Here we note that for the following analysis we need care only about Q_d. Further, we define Q_a as the worker's wage per unit time.

The machine is now working the time m, is serviced the time b, and waiting the time d, per unit of time.

If every worker adjusts n machines, we get the total cost per unit of time:

$$mQ_m + bQ_b + dQ_d + \frac{1}{n} Q_a \quad \text{Eq. 47.}$$

During this time we get the quantity m of manufacture. The cost for this quantity is then mQ_0 and equals the expression in Eq. 47. Then we get:

$$Q_0 = Q_m + \frac{b}{m} Q_b + \frac{d}{m} Q_d + \frac{1}{mn} Q_d \quad \text{Eq. 48.}$$

If we define:

$$V = \frac{Q_d}{Q_a} \quad \text{Eq. 49.}$$

and use the relation $b=km$ we obtain from Eq. 48.:

$$Q_0 = Q_m + kQ_b + Q_a \frac{V_d + \frac{1}{n}}{m} \quad \text{Eq. 50.}$$

For a given machine and a given manufacture k is determined. For different n in this relation only d and m are varying. That is, we are going to minimize the expression:

$$\frac{V_d + \frac{1}{n}}{m} \qquad \text{Eq. 51.}$$

Regarding this derivation, the following can be remarked. Perhaps it would seem reasonable to assume, without an analysis of the problem, that the sum of costs for the idling time, and the worker per unit time, should be minimized. That is, we should then minimize:

$$V_d + \frac{1}{n} \qquad \text{Eq. 51a.}$$

The error we thus commit, is, obviously, that we do not take care of the reduction of the quantity of manufacture caused by the idling time. As it is the cost per product manufactured which is decisive in the economical analysis, it is more correct to work with expression Eq. 51. than Eq. 51a. Further, we remark that we must devote much attention to the calculation of Q_d and Q_a in order to get a true result. We must not only include all real costs but also try not to encumber them with a lot of, with respect to the determination of the optimal number of machines per worker, irrelevant costs. Therefore, perhaps, it should be recommended that the costs, calculated for other purposes, be treated with a certain amount of caution.

The result obtained above, that Eq. 51. shall be minimized, is of course true, independent of the fact that there are one or several workers servicing each group of machines. In this case, however, n always means the number of machines per worker. Here we have not taken into consideration the possibility that the worker, during the time when there is no machine to service, is occupied with other jobs. In this case we get a reduction of the cost per machine. It is easy to correct the expression Eq. 51. in such a case. For special cases like this, which are rather uncommon, it will not pay to construct tables which are generally valid. One should calculate each case separately. We can also find other factors in the calculation of the cost which will complicate the form of Eq. 51. It will probably not be worthwhile to analyze such cases too closely.

The optimal value of n, that is the n value for which Eq. 51. has a minimum, will be called n_0. It is evident that n_0 is a function of two quantities, k and V. Despite this we can, in a relatively simple way, construct tables of n_0 which are dense enough, and we can also obtain a convenient graphical representation. First we will treat the necessary systematization of the calculation work.

In order to indicate to what number of machines per worker the idling and machine times in Eq. 51. are related, we define the self-evident symbols d_n and m_n. If $R_n(V)$ means expression Eq. 51. we obtain:

$$R_n(V) = \frac{Vd_n + \frac{1}{n}}{m_n} \qquad \text{Eq. 52.}$$

If R_n has a minimum for $n = n_0$ this means that:

$$\left.\begin{aligned} R_{n_0-1}(V) &> R_{n_0}(V) \\ R_{n_0+1}(V) &> R_{n_0}(V) \end{aligned}\right\} \qquad \text{Eq. 53.}$$

Assume that this is true for a certain value of V. If V increases it is evident that n_0 must decrease. This means that for some special V value, the second inequality in Eq. 53. must change to an equality. This value, which we call V_{n_0+1} is then obtained from the equation:

$$R_{n_0+1}(V_{n_0+1}) = R_{n_0}(V_{n_0+1}) \qquad \text{Eq. 54.}$$

From this equation we can now compute the *limiting values* V_1, V_2, V_3, etc. These values have the following property. For all values of V between two limiting values, V_{n_0} and V_{n_0+1}, the optimal number of machines per worker is n_0.

The calculation will now proceed as follows. For different, sufficiently dense, values of k, we construct tables of the same type as Table 1. In that way we get machine time and idling time determined as functions of n and k. From Eq. 54. we then get the limiting values V_1, V_2, V_3, etc. These calculations are performed for the same k values as the previous calculations. One limiting value V_n, is now, for a given n, a function of k only. Now we can plot V_1, V_2, V_3, etc. in a diagram as functions of k. This diagram will be referred to as the *optimal diagram*. It is used in the following way. The curves for V_n will divide the diagram into zones. The zone, which is bounded by the curve for V_n and the curve V_{n+1} will be called the nth zone. For all points in this zone, that is, for all V and k values which are represented by points in this zone, the optimal number of machines per worker is n.

In the optimal diagram we have a very convenient method of finding the optimal number of machines per worker without any calculations.

The method described above can be used independently of whether each group of machines is serviced by one or several cooperating workers. However, in the case with several workers we have to construct a diagram for each value of p, the number of cooperating workers. In this case we must notice that in the analysis above n is the number of machines per worker. As it has been, up till now, rather uncommon to have groups of machines serviced by several cooperating workers, we will consider only the case with one worker per group of machines. We will show how the solution of Eq. 54. can be obtained in the simplest possible way.

We want the value of V which satisfies the equation:

$$R_{n+1}(V_{n+1}) = R_n(V_{n+1}) \qquad \text{Eq. 55.}$$

According to Eq. 52. we obtain:

$$\frac{V_{n+1}d_{n+1} + \frac{1}{n+1}}{m_{n+1}} = \frac{V_{n+1}d_n + \frac{1}{n}}{m_n}$$

or

$$V_{n+1}(d_{n+1}m_n - d_n m_{n+1}) = \frac{m_{n+1}}{n} + \frac{m_n}{n+1}$$

If we express d_n and d_{n+1} in m_n and m_{n+1} by the first relation in Eq. 46. we obtain, after a few simplifications:

$$(n+1)V_{n+1} = \frac{m_{n+1}}{n(m_n - m_{n+1})} - 1 \qquad \text{Eq. 56.}$$

By putting $n = 1, 2, 3, \cdots$, and by taking the values of m_n from the tables for a given k, we get V_1, V_2, V_3, etc. In order to show how the calculations proceed, we have collected the results for $k = 0.1$. In this table the m values given in Table 1 are used. (See Table 2.)

The second column in this table gives the limiting values for the V quantities. This means that a certain n value, 9 for example, is optimal for all V values between V_n and V_{n+1}, that is, if $n = 9$, for all V values between 0.461 and 0.295. Let us assume that $V = 1$, that is, that the cost per unit of time for a waiting machine is the same as for a worker. This V value lies between the values 1.20 and 0.74 in the table. Therefore, $n_0 = 7$.

The third column of the table shows the cost factor, Eq. 52., evaluated at the limiting points. In the table we have, for example, the value of 0.402 for $n = 4$. This means, that for a V value for which the optimal n changes from 4 to 5 the cost factor is 0.402. When multiplied by the cost per unit of time for the worker, this factor gives the cost of the servicing and idling times for one unit of manufactured product, that is, for the quantity of product a steadily working machine produces per unit of time.

The last column of the table is included only in order to show how flat the minimum is. Here we have chosen $V = 1$ and calculated the cost factor for different n. The minimum occurs at $n = 7$, which is in agreement with the result of the example above. Because the minimum is rather flat, a change in the number of machines, by one, will give only a few per cent change in the cost. A change in the number of machines by two, however, gives about a 10% increase in the cost. If one estimates so far incorrectly the suitable number of machines that one gets half or double the optimal value, the cost will increase about 100%.

TABLE 2

n	V_n+1	$\frac{R_n}{(V_n+1)}$	$R_n(1)$
1	60	1.100	1.1000
2	17.2	0.711	0.5637
3	7.26	0.516	0.3929
4	3.69	0.402	0.3152
5	2.05	0.325	0.2760
6	1.20	0.271	0.2579
7	0.74	0.210	0.2538
8	0.461	0.201	0.2602
9	0.295	0.178	0.2759
10	0.189	0.160	0.3006
11	0.122	0.146	0.3340
12	0.077	0.134	0.3766
13	0.0491	0.125	0.4289
14	0.0307	0.119	0.4903

ARTICLE NO. 3

In order to facilitate the use of the methods described, certain numerical calculations have been performed and the results have been collected in Table 3. These tables show the idling-time d, the machine-time m, and the productive time of the worker $1-P$, for different values of k and n. For each value of k the calculations have been performed for all n values which give less than 30% idling-time; larger values will scarcely ever occur in practice. We have chosen 14 values of k between 0.01 and 0.40; values outside this range will seldom occur in the applications.

The quantities d, m, and $1-P$ are always numbers between 0 and 1. For the table, however, they have been given in per cents.

From the tables all quantities of interest can be easily calculated. If we choose $k = 0.04$ and $n = 20$, we get for the idling-time 6.4% and the machine-time 90.0%. Then the remaining 3.6% gives the servicing time (according to Eq. 1.). Eq. 2. is satisfied because $3.6/90.0 = 0.04$. The productive time of the worker is 72.0% from the table, which means that he is free from servicing work during 28.0% of the time. Observe that servicing time can also be obtained according to Eq. 36. by dividing the productive time of the worker, 72.0%, by the number of machines, 20.

From the values in the tables we have also constructed an optimal diagram (see Fig. 1). It has turned out to be most convenient to draw the diagram on log-log paper. The abscissa gives k in the range 0.02 to 0.40. On the ordinate V is shown, in the range 0.02 to 100.0. Instructions for use of the optimal diagram were given earlier.

The larger the zone numbers, the closer the zone limits. In order to make the diagram easier to read, the higher zones have been taken together in groups. This is a negligible inconvenience because the cost minimum is very flat for high n values. An error in n of one or two units is therefore of no importance.

As an example we can take $k = 0.045$ and $V = 0.09$. In this case we are in a zone with the designation 21/23. By estimation of the distance to the zone boundaries we find easily that the optimal number of machines is 22, possibly 23.

Because of the scale of the diagram the largest optimal number of machines which can be read in the diagram is 40. In the lower right hand corner of the diagram there is an unused area. This area, which is bounded by a dashed curve, corresponds to idling-times larger than 30%, which, as mentioned earlier, have not been included in the tabulations.

TABLE 3

$k=0.01$

n	d	m	$1-P$
1	0.0	99.0	1.0
2	0.0	99.0	2.0
3	0.0	99.0	3.0
4	0.0	99.0	4.0
5	0.0	99.0	5.0
6	0.1	99.0	5.9
7	0.1	99.0	6.9
8	0.1	98.9	7.9
9	0.1	98.9	8.9
10	0.1	98.9	9.9
11	0.1	98.9	10.9
12	0.1	98.9	11.9
13	0.1	98.9	12.9
14	0.1	98.9	13.8
15	0.2	98.9	14.8
16	0.2	98.8	15.8
17	0.2	98.8	16.8
18	0.2	98.8	17.8
19	0.2	98.8	18.8
20	0.2	98.8	19.8
21	0.2	98.8	20.7
22	0.3	98.8	21.7
23	0.3	98.7	22.7
24	0.3	98.7	23.7
25	0.3	98.7	24.7
26	0.3	98.7	25.7
27	0.3	98.7	26.6
28	0.4	98.7	27.6
29	0.4	98.6	28.6
30	0.4	98.6	29.6
31	0.4	98.6	30.6
32	0.4	98.6	31.6
33	0.4	98.6	32.5
34	0.5	98.6	33.5
35	0.5	98.5	34.5
36	0.5	98.5	35.5
37	0.5	98.5	36.4
38	0.6	98.5	37.4
39	0.6	98.4	38.4
40	0.6	98.4	39.4
41	0.6	98.4	40.3
42	0.7	98.4	41.3
43	0.7	98.3	42.3
44	0.7	98.3	43.3
45	0.7	98.3	44.2
46	0.8	98.3	45.2
47	0.8	98.2	46.2
48	0.8	98.2	47.1
49	0.9	98.2	48.1
50	0.9	98.1	49.1
51	0.9	98.1	50.0
52	1.0	98.1	51.0
53	1.0	98.0	52.0
54	1.0	98.0	52.9
55	1.1	98.0	53.9
56	1.1	97.9	54.8
57	1.1	97.9	55.8
58	1.2	97.8	56.8
59	1.2	97.8	57.7
60	1.3	97.8	58.7
61	1.3	97.7	59.6
62	1.4	97.7	60.6
63	1.4	97.6	61.5
64	1.5	97.6	62.4
65	1.5	97.5	63.4
66	1.6	97.4	64.3
67	1.7	97.4	65.3
68	1.7	97.3	66.2
69	1.8	97.3	67.1
70	1.8	97.2	68.0
71	1.9	97.1	69.0
72	2.0	97.0	69.9
73	2.1	97.0	70.8
74	2.1	96.9	71.7
75	2.2	96.8	72.6
76	2.3	96.7	73.5
77	2.4	96.6	74.4
78	2.5	96.5	75.3
79	2.6	96.4	76.2
80	2.7	96.3	77.1
81	2.8	96.2	77.9
82	3.0	96.1	78.8
83	3.1	96.0	79.6
84	3.2	95.8	80.5
85	3.4	95.7	81.3
86	3.5	95.5	82.2
87	3.7	95.4	83.0
88	3.8	95.2	83.8
89	4.0	95.1	84.6
90	4.2	94.9	85.4
91	4.4	94.7	86.2
92	4.6	94.5	86.9
93	4.8	94.3	87.7
94	5.0	94.1	88.4
95	5.2	93.8	89.1
96	5.5	93.6	89.8
97	5.8	93.3	90.5
98	6.0	93.0	91.2
99	6.3	92.7	91.8
100	6.7	92.4	92.4
101	7.0	92.1	93.0
102	7.3	91.8	93.6
103	7.7	91.4	94.2
104	8.1	91.0	94.7
105	8.5	90.6	95.2
106	8.9	90.2	95.7
107	9.3	89.8	96.1
108	9.8	89.4	96.5
109	10.2	88.9	96.9
110	10.7	88.4	97.3
111	11.2	87.9	97.6
112	11.7	87.4	97.9
113	12.3	86.9	98.2
114	12.8	86.3	98.4
115	13.4	85.8	98.6
116	13.9	85.2	98.8
117	14.5	84.6	99.0
118	15.1	84.1	99.2
119	15.7	83.5	99.3
120	16.3	82.9	99.4
121	16.9	82.3	99.5
122	17.5	81.7	99.6
123	18.1	81.1	99.7
124	18.8	80.4	99.8
125	19.4	79.8	99.8
126	20.0	79.2	99.8
127	20.6	78.6	99.9
128	21.2	78.1	99.9
129	21.8	77.5	99.9
130	22.4	76.9	99.9
131	22.9	76.3	100.0
132	23.5	75.7	100.0
133	24.1	75.2	100.0
134	24.6	74.6	100.0
135	25.2	74.1	100.0
136	25.7	73.5	100.0
137	26.3	73.0	100.0
138	26.8	72.5	100.0
139	27.3	71.9	100.0
140	27.9	71.4	100.0
141	28.4	70.9	100.0
142	28.9	70.4	100.0
143	29.4	69.9	100.0
144	29.9	69.4	100.0

$k=0.02$

n	d	m	$1-P$
1	0.0	98.0	2.0
2	0.0	98.0	3.9
3	0.1	98.0	5.9
4	0.1	97.9	7.8
5	0.2	97.9	9.8
6	0.2	97.8	11.7
7	0.3	97.8	13.7
8	0.3	97.7	15.6
9	0.4	97.7	17.6
10	0.4	97.6	19.5
11	0.5	97.6	21.5
12	0.5	97.5	23.4
13	0.6	97.5	25.3
14	0.6	97.4	27.3
15	0.7	97.4	29.2
16	0.8	97.3	31.1
17	0.9	97.2	33.1
18	0.9	97.1	35.0
19	1.0	97.1	36.9
20	1.1	97.0	38.8
21	1.2	96.9	40.7
22	1.3	96.8	42.6
23	1.4	96.7	44.5
24	1.5	96.6	46.4
25	1.6	96.5	48.3
26	1.7	96.4	50.1
27	1.8	96.3	52.0
28	1.9	96.1	53.8
29	2.1	96.0	55.7
30	2.2	95.9	57.5
31	2.4	95.7	59.3
32	2.5	95.6	61.2
33	2.7	95.4	63.0
34	2.9	95.2	64.7
35	3.1	95.0	66.5
36	3.3	94.8	68.2
37	3.6	94.6	70.0
38	3.8	94.3	71.7
39	4.1	94.1	73.4
40	4.3	93.8	75.0
41	4.7	93.5	76.7
42	5.0	93.2	78.3
43	5.3	92.8	79.8
44	5.7	92.4	81.3
45	6.1	92.0	82.8
46	6.6	91.6	84.3
47	7.1	91.1	85.7
48	7.6	90.6	87.0
49	8.1	90.1	88.3
50	8.7	89.5	89.5
51	9.3	88.9	90.7
52	10.0	88.3	91.8
53	10.7	87.6	92.8
54	11.5	86.8	93.8
55	12.3	86.0	94.6
56	13.1	85.2	95.4
57	14.0	84.3	96.1
58	14.9	83.4	96.8
59	15.9	82.5	97.3
60	16.8	81.5	97.8
61	17.9	80.5	98.3
62	18.9	79.5	98.6
63	19.9	78.5	98.9
64	21.0	77.5	99.2
65	22.0	76.4	99.4
66	23.1	75.4	99.5
67	24.2	74.4	99.6
68	25.2	73.3	99.7
69	26.2	72.3	99.8
70	27.2	71.3	99.9
71	28.2	70.4	99.9
72	29.2	69.4	99.9

$k=0.03$

n	d	m	$1-P$
1	0.0	97.1	2.9
2	0.1	97.0	5.8
3	0.2	96.9	8.7
4	0.3	96.8	11.6
5	0.4	96.7	14.5
6	0.5	96.6	17.4
7	0.6	96.5	20.3
8	0.7	96.4	23.1
9	0.8	96.3	26.0
10	1.0	96.2	28.8
11	1.1	96.0	31.7
12	1.3	95.9	34.5
13	1.4	95.7	37.3
14	1.6	95.5	40.1
15	1.8	95.4	42.9
16	2.0	95.2	45.7
17	2.2	94.9	48.4
18	2.4	94.7	51.2
19	2.7	94.5	53.9
20	3.0	94.2	56.5
21	3.3	93.9	59.2
22	3.6	93.6	61.8
23	3.9	93.3	64.4
24	4.3	92.9	66.9
25	4.7	92.5	69.4
26	5.2	92.1	71.8
27	5.7	91.6	74.2
28	6.2	91.1	76.5
29	6.8	90.5	78.7
30	7.4	89.9	80.9
31	8.1	89.2	83.0
32	8.9	88.5	84.9
33	9.7	87.7	86.8
34	10.6	86.8	88.5
35	11.6	85.9	90.2
36	12.6	84.9	91.6
37	13.7	83.8	93.0
38	14.9	82.6	94.2
39	16.1	81.4	95.3
40	17.4	80.2	96.2
41	18.8	78.9	97.0
42	20.1	77.5	97.7
43	21.6	76.2	98.2
44	23.0	74.8	98.7
45	24.4	73.4	99.0
46	25.9	72.0	99.3
47	27.3	70.6	99.5
48	28.7	69.2	99.7

$k=0.04$

n	d	m	$1-P$
1	0.0	96.2	3.8
2	0.2	96.0	7.7
3	0.3	95.9	11.5
4	0.5	95.7	15.3
5	0.7	95.5	19.1
6	0.9	95.3	22.9
7	1.1	95.1	26.6
8	1.3	94.9	30.4
9	1.5	94.7	34.1
10	1.8	94.4	37.8
11	2.1	94.1	41.4
12	2.4	93.8	45.0
13	2.8	93.5	48.6
14	3.2	93.1	52.1
15	3.6	92.7	55.6
16	4.0	92.3	59.1
17	4.5	91.8	62.4
18	5.1	91.3	65.7
19	5.7	90.7	68.9
20	6.4	90.0	72.0
21	7.1	89.3	75.0
22	8.0	88.5	77.9
23	8.9	87.6	80.6
24	9.9	86.7	83.2
25	11.0	85.6	85.6
26	12.2	84.5	87.9
27	13.4	83.2	89.9
28	14.8	81.9	91.7
29	16.3	80.5	93.3
30	17.9	79.0	94.7
31	19.6	77.4	95.9
32	21.3	75.7	96.9
33	23.0	74.0	97.7
34	24.8	72.3	98.4
35	26.6	70.6	98.8
36	28.4	68.9	99.2
37	30.1	67.2	99.5

$k=0.05$

n	d	m	$1-P$
1	0.0	95.2	4.8
2	0.2	95.0	9.5
3	0.5	91.8	14.2
4	0.7	94.5	18.9
5	1.0	94.3	23.6
6	1.4	94.0	28.2
7	1.7	93.6	32.8
8	2.1	93.3	37.3
9	2.5	92.9	41.8
10	3.0	92.4	46.2
11	3.5	91.9	50.6
12	4.1	91.4	54.8
13	4.7	90.8	59.0
14	5.4	90.1	63.1
15	6.2	89.3	67.0
16	7.1	88.5	70.8
17	8.1	87.6	74.4
18	9.1	86.5	77.9
19	10.4	85.4	81.1
20	11.7	84.1	84.1
21	13.1	82.7	86.9
22	14.7	81.2	89.3
23	16.5	79.6	91.5
24	18.3	77.8	93.4
25	20.2	76.0	95.0
26	22.2	74.1	96.3
27	24.3	72.1	97.3
28	26.4	70.1	98.1
29	28.5	68.1	98.7

$k=0.06$

n	d	m	$1-P$
1	0.0	94.3	5.7
2	0.3	94.0	11.3
3	0.7	93.7	16.9
4	1.1	93.3	22.4
5	1.5	92.9	27.9
6	2.0	92.5	33.3
7	2.5	92.0	38.6
8	3.1	91.4	43.9
9	3.7	90.8	49.0
10	4.5	90.1	54.1
11	5.3	89.4	59.0
12	6.2	88.5	63.7
13	7.3	87.5	68.2
14	8.4	86.4	72.6
15	9.7	85.2	76.6
16	11.2	83.8	80.4
17	12.8	82.3	83.9
18	14.6	80.6	87.0
19	16.5	78.8	89.8
20	18.6	76.8	92.2
21	20.8	74.7	94.1
22	23.1	72.5	95.7
23	25.5	70.3	97.0
24	27.9	68.0	98.0
25	30.3	65.8	98.7

$k=0.07$

n	d	m	$1-P$
1	0.0	93.5	6.5
2	0.4	93.1	13.0
3	0.9	92.6	19.5
4	1.4	92.1	25.8
5	2.0	91.6	32.1
6	2.7	91.0	38.2
7	3.4	90.3	44.2
8	4.3	89.5	50.1
9	5.2	88.6	55.8
10	6.3	87.6	61.3
11	7.5	86.4	66.6
12	8.9	85.1	71.5
13	10.4	83.7	76.2
14	12.2	82.1	80.4
15	14.1	80.3	84.3
16	16.2	78.3	87.7
17	18.5	76.2	90.6
18	21.0	73.9	93.1
19	23.5	71.5	95.1
20	26.2	69.0	96.6
21	28.9	66.5	97.7

$k=0.08$

n	d	m	$1-P$
1	0.0	92.6	7.4
2	0.5	92.1	14.7
3	1.2	91.5	22.0
4	1.9	90.9	29.1
5	2.7	90.1	36.1
6	3.5	89.3	42.9
7	4.5	88.4	49.5
8	5.7	87.3	55.9
9	7.0	86.1	62.0
10	8.5	84.8	67.8
11	10.1	83.2	73.2
12	12.0	81.4	78.2
13	14.2	79.5	82.7
14	16.5	77.3	86.6
15	19.0	75.0	90.0
16	21.8	72.4	92.7
17	24.6	69.8	94.9
18	27.6	67.1	96.6
19	30.5	64.4	97.8

$k=0.09$

n	d	m	$1-P$
1	0.0	91.7	8.3
2	0.7	91.1	16.4
3	1.4	90.4	24.4
4	2.3	89.6	32.3
5	3.3	88.7	39.9
6	4.5	87.7	47.3
7	5.8	86.5	54.5
8	7.3	85.1	61.3
9	9.0	83.5	67.6
10	10.9	81.7	73.6
11	13.1	79.7	78.9
12	15.6	77.5	83.7
13	18.3	75.0	87.8
14	21.2	72.3	91.1
15	24.2	69.5	93.8
16	27.4	66.6	95.9
17	30.6	63.7	97.4

$k=0.10$

n	d	m	$1-P$
1	0.0	90.9	9.1
2	0.8	90.2	18.0
3	1.8	89.3	26.8
4	2.8	88.3	35.3
5	4.1	87.2	43.6
6	5.5	85.9	51.5
7	7.1	84.4	59.1
8	9.0	82.7	66.2
9	11.2	80.8	72.7
10	13.6	78.5	78.5
11	16.3	76.1	83.7
12	19.3	73.4	88.0
13	22.5	70.4	91.6
14	25.9	67.4	94.3
15	29.4	64.2	96.4

$k=0.15$

n	d	m	$1-P$
1	0.0	87.0	13.0
2	1.7	85.5	25.7
3	3.6	83.8	37.7
4	6.0	81.8	49.1
5	8.7	79.4	59.6
6	11.8	76.7	69.0
7	15.4	73.5	77.2
8	19.5	70.0	84.0
9	23.8	66.2	89.4
10	28.4	62.3	93.4

$k=0.20$

n	d	m	$1-P$
1	0.0	83.3	16.7
2	2.7	81.1	32.4
3	5.9	78.4	47.0
4	9.8	75.2	60.2
5	14.2	71.5	71.5
6	19.2	67.4	80.8
7	24.6	62.8	88.0
8	30.3	58.1	93.0

$k=0.30$

n	d	m	$1-P$
1	0.0	76.9	23.1
2	5.1	73.0	43.8
3	11.1	68.4	61.6
4	18.0	63.1	75.7
5	25.4	57.4	86.1
6	33.0	51.6	92.8

$k=0.40$

n	d	m	$1-P$
1	0.0	71.4	28.6
2	7.5	66.0	52.8
3	16.3	59.8	71.8
74	25.6	53.1	85.0
[5	34.9	46.5	93.0

APPENDIX

CALCULATIONS OF THE TRANSIT-TIME

When calculating the optimal number of automatic machines we must include the transit-time in the servicing-times for each machine. The transit-time is defined as the time it takes for a worker to move (with his tools) from one machine to another which requires servicing. This transit-time can be considered as being of the form $a+bd$ where d is the distance the worker has to go. The constant a, which is independent of d, is the time the

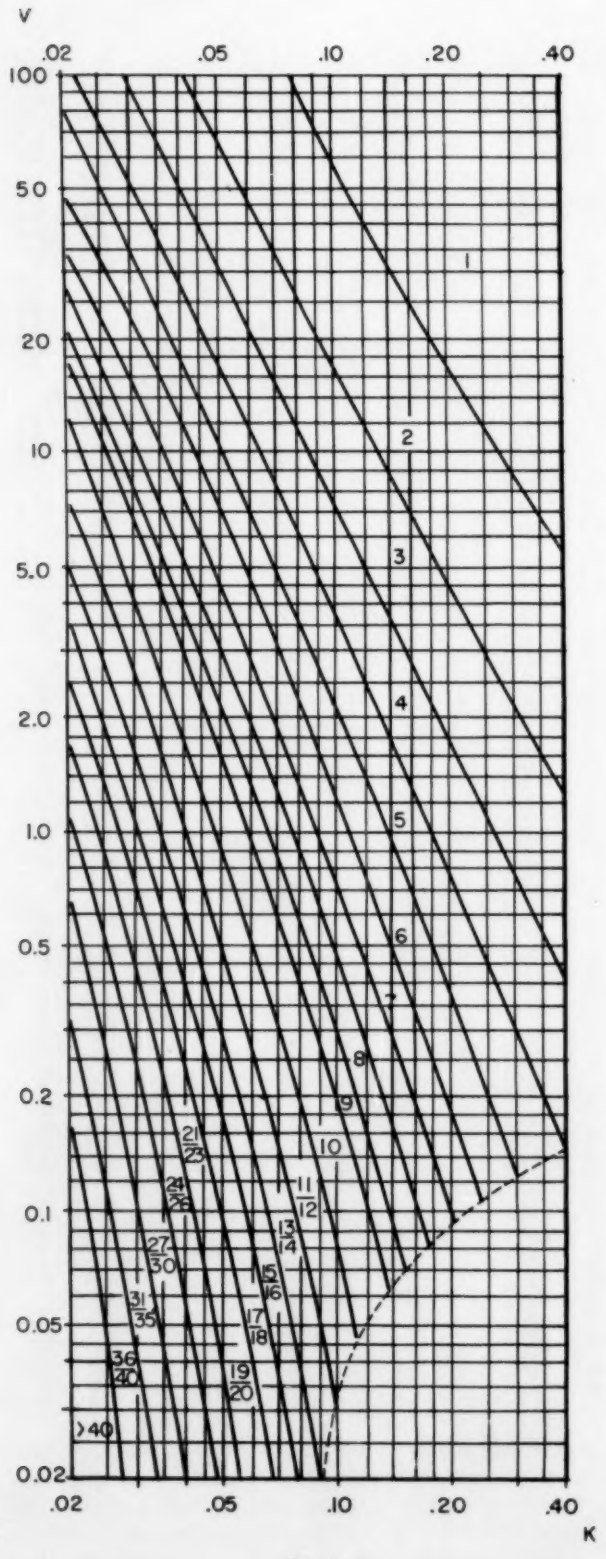

FIG. 1

worker needs to finish the old job and start the new. It should include the time needed to recognize a call for service and the time for taking the correct position at the machine. Even the constant b must be determined by time studies as it is dependent on which tools have to be moved between the machines and also, perhaps, on the practicability of moving the tools. Here we will show how the average of the distance d can be determined in the usual arrangements of the machines, that is, having the machines in one or two parallel rows. In certain cases, for example, when we consider weaving machines, this corresponds to two and four rows, because the machines are arranged in groups, each group consisting of two rows of machines placed so that all the machines can be served from the same straight passage.

First we consider the simplest case with only one row of machines, Fig. 2. The distance between two machines standing beside each other is v and there are n machines in the row. The probability of a call is the same for all the machines and the probability that the worker, when a certain machine gives a call, is stationed at one of the other machines is the same for all machines. This means that all distances (with their direction) between the n machines have the same probability.

For every position (μ) of the worker there are $n-1$ distances. In all there are $n\cdot(n-1)$ distances and it is the average of these distances which we want to calculate. Now there are n different distances, that is v, $2v$, $3v$, $\cdots$, nv.

The distance v occurs twice for every μ except at the end points. In the end positions, where $\mu = 1$ or n, v occurs only once in each position. Thus v will occur $2(n-1)$ times.

The distance $2v$ occurs twice for $\mu = 3, 4, \cdots, n-2$; for $\mu = 1, 2, n-1$ or n it occurs only once. Hence $2v$ will occur:

$$2(n-4)+4 = 2(n-2) \text{ times.}$$

Generally, the distance rv will occur:

$$2(n-r) \text{ times.}$$

This is valid for r up to $n-1$. Larger distances do not exist. As a check of the derivation we can calculate the total number of distances which shall be $n(n-1)$. We find easily that:

$$\sum_{r=1}^{n-1} 2(n-r) = n(n-1)$$

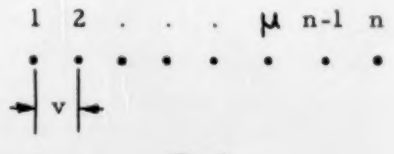

FIG. 2

The average d_1 of all the distances, is now by definition:

$$d_1 = \frac{\sum_{r=1}^{n-1} 2rv(n-r)}{n(n-1)} \quad \text{or}$$

$$d_1 = \frac{2v}{n-1}\sum_{r=1}^{n-1} r - \frac{2v}{n(n-1)}\sum_{r=1}^{n-1} r^2.$$

Now, as one can easily check:

$$\sum_{r=1}^{n-1} r = \frac{n(n-1)}{2}$$

$$\sum_{r=1}^{n-1} r^2 = \frac{n(n-1)(2n-1)}{6}$$

Finally, we obtain:

$$d_1 = \frac{n+1}{3}v$$

which is valid for $n \geq 2$.

The other case, which we will investigate, is when we have two parallel rows of machines with n machines in each row as in Fig. 3. The distance between the rows is v_0; the distance between two neighboring machines in the same row is v.

For the distance in each row taken separately Eq. 1. is valid. The average of the distances from the machines in one row to the machines in the second is, however, somewhat different. First, if we take the shortest distances between the machines we will get an expression of the form:

$$\sqrt{v_0^2 + r^2v^2}$$

for the distances.

The average is then a sum of such expressions, r having the values $0, 1, 2, \cdots, (n-1)$. For this sum it seems impossible to obtain a simplified, numerically convenient expression. Instead, one has to calculate each square root separately, which is rather laborious. However, by a different formulation of the problem, which actually is closer to the true state of affairs, we can avoid these difficulties. Because the machines will often block the diagonal paths it seems natural to assume that the worker moves only parallel and at right angles to the rows of machines. Then we get expressions of the form:

$$v_0 + rv$$

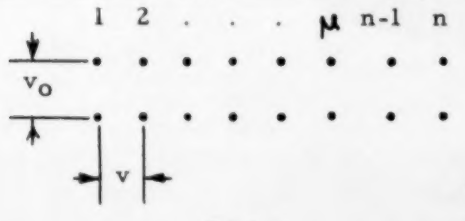

FIG. 3

instead of the square roots. As shown below we then get a group of series expressions which may easily be put into closed form.

Depending on the symmetry between the rows, it is sufficient to consider the average of the distances from a machine in the first row to the machines in the second row, according to Fig. 3. The number of such distances is evidently n^2. In a way, similar to that used before, we then find that:

The distance v_0 occurs n times.
The distance v_0+v occurs $2\ (n-1)$ times.
The distance v_0+2v occurs $2(n-2)$ times, and so on, so that generally:
The distance v_0+rv occurs $2(n-r)$ times.
The total number of distances is now obtained from:

$$n + \sum_{r=1}^{n-1} 2(n-r)$$

which we find equals n^2 as it should. The average of all the distances is then:

$$d_2 = \frac{1}{n^2}\left\{nv_0 + \sum_{r=1}^{n-1} 2(n-r)(v_0+rv)\right\} \qquad \text{Eq. 57.}$$

which can be simplified to:

$$d_2 = v_0 + \frac{n^2-1}{3n}v \qquad \text{Eq. 58.}$$

In order to obtain the average value d for all the distances between the $2n$ machines in Fig. 3, we will finally compute the average values of d_1 and d_2. We note that d_1 is valid for $n(n-1)$ distances while d_2 is valid for n^2 distances. Then we obtain:

$$d = \frac{n(n-1)d_1 + n^2d_2}{n(n-1)+n^2}$$

and with Eq. 57. and Eq. 58.

$$d = \frac{n}{2n-1}v_0 + \frac{2}{3}\cdot\frac{n^2-1}{2n-1}v \qquad \text{Eq. 59.}$$

For large values of n, d_1 will approach $nv/3$, d_2 will approach $v_0+nv/3$ and d will approach $v_0/2+nv/3$.

As mentioned earlier, cases can occur when two rows of machines can be considered as only one row, because the transit times between opposite machines can be neglected. The expression above must then be revised because the worker can get a call for service from the machine opposite the machine with which he is occupied. When we have an arrangement as in Fig. 2, where each dot now represents two machines, we obtain the revised average d_1' directly from Eq. 59. by putting $v_0=0$. We obtain:

$$d_1' = \frac{2}{3}\cdot\frac{n^2-1}{2n-1}v \qquad \text{Eq. 60.}$$

If the arrangement is according to Fig. 3, where each dot now represents two machines, we can reason in the following way: The average distance between one machine and the other machine in the same row is given in Eq. 60. The number of distances in this row is then $2n^2-n$. The average of the distances between one machine and the machines in the other row is given in Eq. 58. and the number of distances of this type is $2n^2$. The average of all the distances is then:

$$d' = \frac{(2n^2 - n)d_1' + 2n^2 d_2}{4n^2 - n}$$

and so:

$$d' = \frac{2n}{4 \cdot n - 1} v_0 + \frac{4}{3} \cdot \frac{n^2 - 1}{4n - 1} v \qquad \text{Eq. 61.}$$

For normal values of n, the difference between d' and d is very slight. So also is the difference between d_1' and d_1.

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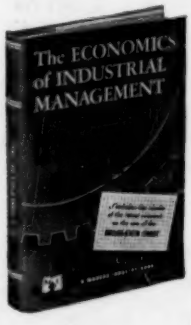

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The Gap Between Theory and Practice in Equipment Replacement

by **RAYMOND R. MAYER**

Assistant Professor of Production, The University of Chicago

A SURVEY of the literature available on the subject of equipment policy indicates that a major portion of it falls into two categories. The first of these includes what can be looked upon as a criticism of existing methods of comparing equipment investment alternatives. These methods consist of the traditional or conventional short-payoff demands, high rate of return requirements, and the like. As opposed to these contributions of a relatively negative nature, the second category is composed of the literature in which there are formulated various techniques that can be used under stipulated conditions to compare capital investment alternatives. These techniques are exemplified by George Terborgh's MAPI formula, Eugene Grant's uniform annual cost comparison, and Joel Dean's discounted cash flow method.

With no intention of minimizing the importance of the contributions of these categories of research, it has been noted that rarely has a case been presented in defense of those firms which have not accepted in their entirety the techniques proposed for use by the theorists. The major criticism levelled by business against the theorists is that the latter are guilty of oversimplifying the problems inherent in any equipment replacement decision. While admitting that the concepts underlying the mathematical models developed for the solution of these problems are sound, industry points out that many of the factors incorporated in these models cannot be expressed in quantitative terms. In other words, a correct method is available but, unfortunately, the data on which the efficacy of this method is dependent are not available. The validity of this criticism can be demonstrated by means of an actual case study of the equipment policy adhered to by a typical manufacturing concern, The Ingersoll Milling Machine Company of Rockford, Illinois.[1] The nature of this policy has been influenced by the problems which confronted the firm in its attempt to develop a rational approach to equipment replacement.

[1] The equipment policy of The Ingersoll Milling Machine Company was studied and analyzed by the author under the terms of an Ingersoll Foundation Fellowship awarded by the National Center of Education and Research in Dynamic Equipment Policy at Illinois Institute of Technology. This paper is based on a portion of the results of his investigation.

THE COMPANY'S OPERATING CHARACTERISTICS

The Ingersoll Milling Machine Company was founded in Cleveland where it was incorporated under the laws of Ohio in 1890. Financial difficulties provided the incentive to move to Rockford, Illinois, in 1891, where operations were resumed in a 50′ × 150′ shop with a labor force of 19 men. Today, operations continue at this site in a plant consisting of 300,000 square feet and employing approximately 1,200 individuals.

As the company's name indicates, at the time of its founding, one of the firm's main products was milling machines. Some time later, its product line was expanded to include cutter grinders, cutters, and fixtures. By the mid-1930's, multi-station process machines were introduced and, in a few years, accounted for an appreciable portion of the company's sales. But the distinguishing characteristic of the company's products is that most of its machines are special rather than standard. These machines are designed to satisfy a particular customer's production requirements. The usual procedure is for the customer to acquaint Ingersoll's engineers with a specific production problem, following which an analysis is made of the customer's work and production requirements. A special machine is then designed whose characteristics are believed to be such that they will permit performance of the required operations at a minimum cost. A proposal is then submitted to the customer in which a fixed price is stipulated, a delivery date promised, and production times guaranteed.

This concentration on special machines has resulted in a number of operating characteristics with which the company can be identified. First, manufacturing operations are non-repetitive. Any one machine being built is rarely an exact duplicate of a machine built earlier. As a result, lot-sizes are small, mass production techniques cannot be employed, and the full benefits of labor specialization cannot be realized. This has created a need for the highest caliber of management, engineering, and production personnel. A second characteristic is that the company is always in competition with manufacturers of standard machine tools having a lower first cost and an adaptability to a variety of tasks which create an ever-present obstacle to be overcome. Consequently, the com-

pany's business is inherently at least as cyclical as that of the machine tool industry as a whole.

Yet, in spite of these difficulties, Ingersoll's history is one of upward trends in sales, employment, and net worth. Average returns on investment have been satisfactory and reflect sound managerial policies. At the present time, the company is characterized by annual sales of abouty twenty million dollars. Its labor force and physical facilities are being expanded, and its financial position is stronger than it ever has been in the past. The company is a recognized leader in its field, and all its current policies have been devised to maintain and strengthen that position.

One of the policies Ingersoll's management considers to be of utmost importance is the company's equipment policy. It bases this belief on the realization that, to maintain and improve its competitive position, the company must, first, have the production facilities to build the machines required by its customers and, second, be able to build those machines at the lowest possible cost. Naturally, the nature of the company's machine tools, as determined by its equipment policy, will have an important effect on the firm's ability to satisfy these two requirements. To begin with, the organization's production capacity will be dependent, to an appreciable degree, on the nature of its production equipment. But just as important, the quality of the organization's production equipment will have a significant influence on unit production costs through its effect on required labor, material, scrap, rework, power, floor space, work-in-process, maintenance, and capital. Unless these are kept to a minimum, the company's sales and profits may suffer.

INAUGURATING THE REPLACEMENT STUDY

The company's methods department has been given the responsibility of bringing to the attention of management the opportunity for making a profitable investment in new equipment. It performs this function by various means. First, it has been instructed to assume that every machine ten years of age or older is a likely candidate for replacement. Consequently, the performance of every machine falling into this category is analyzed once a year with a view towards possible replacement, and the results of these studies are submitted to management. Second, a special repair record maintained for each machine is reviewed periodically, and any machines whose repair costs appear to be excessive are studied for possible replacement. Third, machine operators, shop supervisors, and machine repairmen are instructed to notify the methods analyst of any problems being encountered which are attributable to the unsatisfactory performance of a particular unit of production equipment. For example, the operator may find it difficult to maintain specified tolerances, the supervisor may find it impossible to meet production schedules, or the machine repairman may receive a request for major rehabilitation of a machine, the value of which is highly questionable. Fourth, and last, members of the methods department are expected to keep informed on new developments in machinery design and construction through contacts with machine tool salesmen and by reading current technical publications, attending machine tool exhibits, and visiting machine tool manufacturers. The acquired knowledge of available machines often suggests a possible replacement of an existing unit.

After having determined that consideration should be given to the replacement of a particular machine, the methods analyst selects possible alternatives. One such alternative is rebuilding the machine. The other is procuring a new machine. Potential suppliers of new equipment are selected on the basis of the analysts' knowledge of the manufacturers of various types of machine tools and by referring to available directories. The purchasing department is asked to obtain information regarding new equipment from the manufacturers specified by the methods analyst, and the determination of available alternatives begins.

Insofar as describing the respective alternatives is concerned, the methods department is instructed to concentrate on the direct labor saving made possible by the new machine. However, since the nature of the company's operations is such that no one machine can be associated with any one product, savings in total operating time can be ascertained only on the basis of savings to be realized in the production of typical parts. Certain parts are typical in the sense that they are representative of parts that have been produced in the past and will be produced in the future. The time saving expected on a typical part is assumed to represent the saving which will be realized in the production of every part in the category of work represented by that typical part.

Typical parts for any one machine are selected through the cooperative efforts of the methods supervisor, superintendent, and foreman. After the parts have been selected, the time required to process each of these parts on the old machine is determined by the methods analyst by stop-watch time studies or from historical records on file in the accounting department. Operating times for these same parts on the new machines are obtained from the manufacturers of the machines being considered as possible replacements. Each manufacturer is required to guarantee the estimated operating times he submits.

This is followed by the calculation of an average per cent saving, weighted to reflect the proportion of total machine time spent in the production of each category of parts. The average per cent saving is then applied to the hours the old machine was in operation during the past year to obtain the expected annual saving in hours. This figure, when multiplied by appropriate hourly labor rates, yields the direct labor saving in dollars. On the basis of this saving relative to the machine's first cost, the methods department will select the best alternative.

Completion of these calculations is followed by the preparation of a summary report of the analysis. One section of the report is devoted to the old machine and contains the following information:

Description
Location
Age
First Cost
Book Value
Market Value
Annual Hours of Operation
Operations Performed
Annual Repair Costs

The second section of the report is devoted to the best available alternative machine. It contains the following information:

Machine Description
First Cost
Annual Direct Labor Savings
Other Alternatives Considered
Other Relevant Factors

Other relevant factors may include the need for additional production capacity, extensive overhaul requirements for the old machine, and the like. But more often than not, the only factor considered by the methods analyst is the direct labor saving.

All such studies made on machines ten years of age or older are then sent to the president of the company for his review as a matter of routine. In all other cases, an equipment replacement committee decides whether the summary report should be forwarded to the chief executive. This committee consists of the works manager, the assistant to the vice president of sales, and the methods supervisor. As a rough guide, they compare the estimated life of the machine with the number of years required to recover its first cost by the direct labor saving. For purposes of service life estimating, the company places its machine tools into two categories—"small machines" and "large machines." Based upon its experience, Ingersoll has found that the average economic life of its "small machines" has been ten years and that for the "large machines," it has been fifteen years. One of these figures is accepted as the expected service life of the new machine depending on the category into which the equipment falls. However, there is a realization that although the average life may be, for example, ten years, any one unit in that group may have an actual life which is longer or shorter than this average. Ingersoll management feels, however, that no precise estimate can be made of service life in any one case. The period during which a new machine will be kept in operation will be determined by the degree to which the equipment deteriorates and becomes obsolete. The rate of deterioration will be determined by the physical construction of the machine, the type of operation for which it will be used, the number of hours per year it will be in operation, and the like. The rate of obsolescence will be determined by the future developments in machine tools and by the future demand for the product the equipment is capable of producing. Since there is no way of predicting the nature of these factors, the company chooses to work with probable lives for each of its two classes of equipment.

In any event, if the recovery period is less than or equal to the estimated life, the proposal is sent to the president. However, a recovery period which exceeds the estimated life does not preclude forwarding the study. In those cases, the committee will take into consideration other relevant factors. For example, if the shop needs the additional production capacity made available by the new machine, recovery periods are ignored completely. Therefore, no rigid criteria exist by which the final decision is made. Judgment plays an important role.

THE ROLE OF THE PRESIDENT

The reason for sending the equipment replacement proposal to the president is that, at Ingersoll, the chief executive assumes full responsibility for the decision to buy new equipment. This practice evolved from the belief that the nature of many of the factors to be taken into consideration is such that the president is in the best position to evaluate them correctly. The information made available to him with reference to any one machine being studied for replacement is designed to aid him in making a decision in that particular case. And even the nature of the information he requires from other departments in the organization is specified by the president. To evaluate this concept of the president's role, it will be necessary to discuss the factors the company considers to be relevant and the manner in which they enter the analysis.

Mention was made of the fact that management considers one of the contributions of efficient equipment to be its ability to reduce operating costs. This infers that in any comparison between equipment alternatives, consideration must be given not only to direct labor costs but to indirect labor, routine maintenance, special repairs, power consumption, floor space, subcontract costs, tool costs, supplies, rework, downtime, fringe labor costs, scrap, property taxes, and insurance. A comparison of these elements with the data made available by the methods department to the president reveals that, as a rule, no estimates are made of most of these items. Instead, the emphasis is placed on direct labor costs. The cases in which other costs are mentioned are the exception. On the surface, this appears to be contrary to an approach suggested by management's realization that other cost items will be affected by the nature of the final decision. However, further investigation reveals that those other items are not ignored, although direct labor costs remain the most important cost in replacement studies.

The emphasis is placed on direct labor cost, because it is the most tangible of all costs. To begin with, the

company can estimate the magnitude of this cost if the old machine is retained. Second, insistence on guaranteed production times provides a firm basis on which estimates can be made of the magnitude of this cost if a new machine is procured. However, Ingersoll management does not feel that the same can be said for the other individual cost elements. This is true for one or both of the following reasons. First, it is impossible to estimate accurately the past level of these expenses for any one machine. Second, it is impossible to forecast accurately the future level of these expenses if the old machine is replaced. This can be illustrated using routine maintenance as an example. Routine maintenance is a very straightforward operation, and theoretically the amount of labor and material expended in maintaining a given machine can be measured. But any procedure developed to provide this information would entail so much administrative and clerical time that the value of the data would be more than offset by the cost of procuring them. And even if this were not true, a knowledge of this charge against the old machine would still leave the need for estimating maintenance costs of the proposed replacement machine. Unlike direct labor costs, there is no sound basis on which this estimate can be made. At best, therefore, this cost can be known for the old machine and estimated for the new; at worst, estimates would have to be made for both. But in any case, Ingersoll management feels that the absence of reliable guides minimizes the validity of any such estimate of the future.

Special repairs is taken as a second example. A record is maintained of annual repair costs for each machine in the plant. Thus, in this instance, special repair costs on the old machine are known. And yet, management does not believe that a knowledge of past charges is any guide to an estimate of this expense for the new machine. In addition to the fact that any one machine is almost never replaced by a like machine, past experience indicates that repairs on two identical machines will vary widely and that repair costs have no particular trend. Once again, a sound guide for making future estimates is lacking.

The special difficulties encountered in making estimates of these particular costs are not to be misinterpreted as stemming from the fact that the future rate of production, prices, and the like may vary. They are independent of forces such as these. Ingersoll management would be willing to make the same assumptions for these items as it has made for direct labor, if by doing so it would then be in a position to make sound estimates; savings in direct labor are computed on the basis of last year's production and current labor rates. However, this would do nothing to solve the problem.

In spite of its belief that it is usually impracticable to place a specific dollar value on savings other than direct labor, management does not advocate ignoring these potential benefits when making a cost comparison. To do so is to overlook the fact that a decrease in direct labor personnel will usually be followed by decreases in administrative costs, supervision, fringe benefits, and required employee service facilities. Also, it is to fail to realize that the machine itself may reduce scrap, rework, indirect labor, floor space, and tool costs. On the other hand, it may cause the company to lose sight of the fact that a new asset may increase power consumption, property taxes, and insurance. After careful study of the problem, the firm's management decided that some approximation must be made of the average saving which could be expected on these overhead costs. On the basis of the judgment and experience of operating and administrative executives, it was decided to assume that the decrease in overhead costs would be equivalent to 100 per cent of the anticipated direct labor saving for any one machine.

Acceptance of this assumption is not accompanied by the belief that it is realistic in any one case. On the contrary, it is realized that for any one machine savings may be greater or less than those indicated by the assumption. However, for a number of machines, average overhead savings are believed to equal the savings in direct labor. As a result, Ingersoll management feels that it has succeeded in taking all relevant operating costs into consideration and yet has not adopted an approach which would lead one to believe that precise estimates have been made of factors which are inherently unpredictable to a large degree.

It was mentioned that a review of the summary report prepared by the methods department indicates that the methods analyst does not ordinarily concern himself with operating costs other than direct labor, although the methods analyst will occasionally include some savings, other than direct labor, in his summary report. These savings consist of those items that are predictable in a specific case. However, this is the exception rather than the rule. Instead it is the president who, after receiving the equipment replacement proposal, doubles the estimated direct labor saving to arrive at the approximate total reduction in annual operating costs. The fact that only the chief executive makes use of the "100 per cent" guide insures that it will be employed with discretion. If in any one case he has reason to suspect that the assumption is completely unrealistic, he may question the works manager or the methods supervisor with a view toward making a more accurate estimate. On other occasions, the methods supervisor may be called in to explain the manner in which typical parts were selected, to describe the physical condition of the old machine, to explain extremely large repair costs, to investigate capacity requirements in the shop, and the like.

After having satisfied himself that the data compiled by the methods department are satisfactory and complete, the president goes on to a study of other factors. These are the factors whose evaluation, as stated by the

president, ". . . involves the possession of information and a point of view which by the nature of things are seldom had by the operating technician."

IRREDUCIBLE FACTORS

As would be expected, the computation of future operating costs with which a new machine can be identified calls for a decision regarding the level of operations and prices throughout the life of the machine. It was seen that the methods department, in accordance with management's instructions, assumed, first, that the rate of production on a new machine would be equal to that on the old machine during the year preceding the replacement analysis. Second, direct labor savings were determined by using hourly wage rates in effect at the time the study was being made. This was then followed by the president's approximation of what the savings in overhead costs would be. These savings also were estimated on the basis of current prices and rates of output. This is not to say, however, that the firm's chief executive believes that no changes will take place in the level of operations and prices during the service life of the asset. Instead, the assumptions are the result of a realization that it is impossible to predict accurately the magnitude of these factors for a future period of time ranging from ten to fifteen years. This is especially true of the future level of operations because of the wide fluctuations in demand for the company's product. Consequently, the preliminary analysis made by the methods analyst is based on the supposition that present conditions reflect those of the future.

The president of the company realizes, however, that probable changes in the future cannot be ignored. Hence, in the final analysis, for which he is responsible, data made available to him by the methods department are interpreted in light of his expectations of the future. As a result, a decision regarding the acquisition of new equipment is based on a number of factors other than those taken into consideration by the methods analyst. The nature of these factors is such that they cannot be presently reduced to a dollar advantage or disadvantage. One of these factors is anticipated changes in the level of operations.

The magnitude of savings suggested by the methods department's summary report on a particular machine can be radically affected by changes in the degree of machine utilization. For example, if the computations were based on a two-shift operation, a change to one-shift would reduce anticipated savings in direct labor by 50 per cent. Conversely, inauguration of a three-shift schedule would increase these savings by 50 per cent. In many cases, this would alter the final decision. Therefore, the president must evaluate the data incorporated in the equipment replacement proposal accordingly. If, as a result of his judgment and business experience, he believes that the future level of operations will be higher, he will be inclined to favor the acquisition of new equipment. On the other hand, if the opposite is true, procurement of new production facilities is discouraged.

A second factor affecting the desirability of machine tool replacement is price trends insofar as they affect the magnitude of manufacturing costs. A future rise in wage rates, material costs, and the like would make the savings determined by the methods analyst conservative, and machine tool replacement is encouraged. An anticipated decrease in the cost of these factors of production would tend to exaggerate the level of savings, and the acquisition of new equipment becomes less profitable than indicated. Therefore, in evaluating the benefits of the proposal, the president is influenced by the direction he expects future prices to take.

Price trends are also important as they relate to the purchase cost of new equipment. If there were reason to believe that the cost of the machine being considered for purchase will decrease in the near future, acquisition might be postponed. A contemplated increase in the price of the machine would encourage its acquisition.

Another area in which price trends sometimes play a decisive role is in that related to the salvage value of old equipment. Replacement of existing equipment at Ingersoll is encouraged when its salvage value is high relative to its first cost in spite of the asset's having been used for an appreciable period of time. Retention of the machine may result in a significant decrease in its market value in the near future, and hence a new machine, in addition to providing reduced operating costs, may make possible a capital gain which is subject to a tax of only 25 per cent. For this reason, the president requires that he be furnished with the first cost, book value, and market value of the old equipment. The market value, when compared to the first cost, may reveal the presence of an extraordinarily high market for used equipment. The book value, when compared to the market value, permits determination of the capital gains tax and the resultant net gain to the company.

A third factor taken into consideration by the company's president is the present need for a new machine's increased production potential. Although this is somewhat related to the first factor, which dealt with anticipated changes in the level of operations, the emphasis is placed on current needs as opposed to those that may prevail in the more distant future. In all cases in which the shop is in serious need of additional production capacity, Ingersoll's president feels impelled to provide this capacity by replacing deteriorated and obsolete machine tools with modern and more efficient equipment. Savings in direct labor and overhead expenses, calculated on the basis of the output represented by the old equipment, become of secondary importance. Of course, there are exceptions to this general rule. For example, it may very well be that additional capacity will be required for a period of only a few weeks or months. Naturally, subcon-

tracting would be considered. But where it appears that the condition will prevail for some time, the company's chief executive is inclined to make the necessary additions to the shop's production facilities.

Ordinarily, no comparison is made between the revenue made possible by the additional business and the costs that will be incurred as a result of the increase in sales. Management feels, first, that accurate estimates of this type are almost impossible to make and, second, that if the firm wants to stay in the manufacturing business, the required capacity must be provided. The assumption is made that if the over-all rate of return on the business has been satisfactory more of the same kind of business will also yield satisfactory profits. In other words, once management has decided to remain in business, steps must be taken to satisfy all the requirements of that business. Although the alternative of subcontracting is resorted to on occasion, it is rarely considered satisfactory as a long-term proposition for any one machining operation. The nature of the company's operations makes quality production and prompt deliveries of primary importance; management believes that these conditions can best be satisfied in its own shop.

The fourth major factor considered by the president in making an equipment procurement decision is the availability of capital relative to the company's demand for capital. Ingersoll, just as any other industrial organization, is always confronted by opportunities to invest in various types of capital goods. As opposed to an expenditure on machine tools, money can be spent, for example, on the procurement of land for future plant expansion; or funds can be expended on new buildings or on the modernization of existing buildings. In addition to these opportunities to invest in other fixed assets, there may be an opportunity or a need to invest money in current assets. Increasing sales may be accompanied by a need to increase raw material, purchased parts, and work-in-process inventories; also accounts receivable may increase and thereby reduce the amounts of funds available for capital goods expenditures. All this is kept in mind by the president at the time he is confronted by a proposal calling for the purchase of a new machine tool, and the savings set forth in the proposal are evaluated by him accordingly. The role played by judgment and experience in this activity would be difficult to overemphasize.

Therefore, upon receipt of the results of a replacement study by the methods analyst, the president takes into consideration, in addition to all the other factors already discussed and those to be discussed, the amount of capital available from internal and external sources. With regard to external funds, the terms on which various amounts of money can be obtained are given close attention. Next, the other possible uses for these funds are reflected upon. On many occasions, the supply of capital is more than adequate for existing investment opportunities and the decision is based on other considerations. But in those cases where this is not true, the president studies the savings made possible by a new machine in light of the returns he estimates can be realized from other investments. As has already been mentioned, for this he relies primarily on his judgment and years of experience in the industry. The result is that the level of savings required to justify the procurement of new production equipment may vary not only from month to month but from week to week. It is important to point out that there is no type of capital expenditures budget in effect at the company. No fixed amount is established at the beginning of any period to govern the amount of money to be spent on capital goods during that period. Unlike the practice in some companies, no effort is made to relate these expenditures with available depreciation reserves, past profits, or sales. Instead, the amount to be spent is determined primarily by the potential savings and additional revenues to be realized by investing available funds.

In addition to the four factors discussed above, the president concentrates on a number of miscellaneous factors in special cases. The first of these is related to the nature of Ingersoll's future products. Mention has already been made of the fact that a replacement is often made because of the capacity, in terms of volume of output, that the new machine makes possible. Similarly, a replacement is often encouraged because of a new machine's capacity in terms of the tolerances it can maintain and the size of parts it can process. Whether this type of capacity is valuable depends on the nature of the company's future products.

Two other factors are often relevant in those cases in which a machine is to be replaced by equipment to be built in Ingersoll's own shop. Prior to mentioning these factors, it may be best to point out that if an old machine is to be replaced by one which is in the company's product line, Ingersoll assumes the same role as any other potential supplier. The methods department submits prints of typical parts to the sales department, asks for a quotation, and requests guaranteed production times. One minor difference is that the president, when evaluating his own sales department's proposal, will sometimes give consideration to two factors which are of no consequence when another manufacturer's equipment is being proposed. The first of these is the fact that the order may be of some experimental value to the company. In building a machine for its own use, Ingersoll is able to incorporate new and unique design features which have as yet been untried in an actual production machine. Any resultant production and operating difficulties can be noted and eliminated, and the features are then ready to be built into equipment being manufactured for customers. A second factor, somewhat related to the first, is that the company's sales representatives can utilize these machines of the latest design to demonstrate to cus-

tomers the production potential of Ingersoll's products. The receipt of an order often depends on the company's ability to show in its own shop what a proposed machine is capable of accomplishing.

GENERAL NATURE OF THE JUSTIFICATION

In summary, therefore, it can be said that a number of factors serve as the basis on which the procurement of new equipment is justified at Ingersoll. Yet, it is worth noting that in many cases the president finds it unnecessary to take all these factors into consideration. Often, just the elements evaluated by the methods analyst are sufficient to justify replacement. For example, in one case a new upright drill having a first cost of $5,800 was found to be capable of annual savings of $2,100 in direct labor alone. As a second illustration, a boring machine whose first cost was $14,800 was able to reduce annual direct labor costs by $4,300. In these cases, a more comprehensive analysis was considered unnecessary. But the average replacement study requires a weighing of many elements. One would expect this because of Ingersoll's policy of reviewing its machine tools frequently and in a systematic manner. Equipment is not allowed to deteriorate or become obsolete to a degree that results in unduly excessive operating costs. As a result, the so-called irreducible factors must be examined by the president.

The presence of these factors and management's belief that they cannot be described quantitatively have resulted in the absence of a formulated approach to equipment replacement and acquisition at Ingersoll. This does not mean that management has not investigated the feasibility of adopting other approaches. As new techniques for making analyses of this type have been developed and publicized, the company has studied the principles on which these various methods are based. To date, it has been found that, because of the nature of Ingersoll's operations, none of these methods is completely satisfactory. All refined techniques, designed to aid management in decision making in this area, call for an analysis which involves estimating:

1. The rate or production throughout the life of the new machine.
2. Magnitude of the difference in individual cost and revenue elements between the investment alternatives.
3. Service lives of the alternatives with due consideration to deterioration and obsolescence.
4. Residual value of each of the machines at the end of its service life.
5. The time value of money to the company.

After these estimates have been made, the firm is then able to calculate the rate of return on its investment or the annual cost of each alternative depending on the nature of the specific method employed.

In evaluating each of these methods, Ingersoll management concluded that the non-repetitive nature of the company's operations, the changing nature of its products, and the wide fluctuations in its annual shipments prohibited it from making the precise estimates which were required if any formula were to be used correctly and yield meaningful results. Since these estimates could not be made, it was believed that there was little, if any, reason for management concerning itself with such refinements as compound interest, exact rates of deterioration and obsolescence, values for individual overhead expenses, and the like. As a result, no formulated approach has ever been adopted.

For these same reasons, Ingersoll has never established a payoff period requirement. In this respect, its practice is unlike that of most firms which do not employ refined techniques; the absence of these techniques is often accompanied by the demand that a machine pay for itself in a stipulated number of years from direct labor or other savings it makes possible. At Ingersoll, no such standard has ever been in effect. It is realized that even an approximate requirement of this type infers that all factors to be taken into consideration can be expressed quantitatively. For example, a firm that requires that a new machine pay for itself in five years from direct labor and overhead savings is actually stating that these savings over the coming five-year period can be accurately estimated, that its future capital supply and capital demand will be such that a five-year pay-off represents a satisfactory return, that savings will continue sufficiently beyond the five-year period to make the investment profitable, and so on. Ingersoll management finds these implications, inherent in any standard pay-off or rate-of-return requirement, impossible to accept. Naturally, if factors other than direct labor and overhead savings based on the current level of operations were inconsequential, some reasonable pay-off would be demanded. A fifteen-year pay-off, for example, would be unsatisfactory if the machine were expected to have a service life of only ten years. But the company considers this to be a hypothetical case. Management believes that other relevant elements invariably exist, and there would be no point in specifying a standard requirement for an improbable set of conditions. The result has been the development of an approach designed to satisfy the conditions unique to the company's operations.

In summary, this approach calls for reliable estimates by the methods analyst of the direct labor savings to be expected from a new machine if business continues at the current level. Reductions in overhead costs are then estimated on the basis of expected direct labor savings; ordinarily, it is assumed that savings in overhead will equal savings in direct labor. The president of the company then evaluates these savings in light of other factors. These include:

1. Anticipated changes in the level of operations.
2. Price trends insofar as they may affect the salvage value of the old equipment, purchase cost of new equipment, and magnitude of manufacturing costs.

3. Need for the increased production potential of a new machine.

4. Availability of capital relative to the company's need for capital.

5. Nature of future products.

The best that can be said is that these factors are considered. Their nature determines whether replacement is encouraged or discouraged. However, the degree to which they affect the decision is determined solely on the basis of management's judgment with regard to the direction and intensity with which they act. The belief is that, although no one is in a position to say that one thing or another will definitely happen, years of background in the industry places the firm's chief executive in the most favorable position to make the required assumptions. In any one case, the combination of all these factors determines whether the outlook for the future warrants an optimistic or pessimistic approach to the problem.

THE CASE'S CONTRIBUTION

It can be seen, therefore, that The Ingersoll Milling Machine Company can best be described as a firm which has accepted the principles of equipment replacement theory but has found it necessary to modify the techniques proposed for use by the theorists. The company's management has rejected the suggestion that a formulated approach be adopted in this area of decision-making. The use of a formulated approach would make its intended contribution only if fairly precise estimates could be made of the future. Since Ingersoll management believes that making many of the estimates with the required degree of accuracy is practically impossible, it relies to a relatively high degree on judgment and experience. The arguments it presents in defense of its position appear to be sound and serve, first, to identify areas in which additional research is required and, second, to explain the reluctance of many organizations to adopt one of the many available "scientific" approaches to equipment acquisition and replacement.

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TWO common tools utilized to evaluate the economic potential of alternative ways of performing a specified task are (A) Break-Even Point Analysis, and (B) Economic Lot-Size Theory.

These two techniques have become the basic devices around which many courses in Engineering Economics have been built over the past 40 years. Most engineering undergraduate students at some time are exposed to these two fundamental methods. Industrial Engineers, in particular, find a persistent recurrence of problems that utilize one or the other of these models in their solution.

In retrospect one wonders why these two schemes have never been combined into one common model. On the other hand, they have primarily been developed to solve two problems that at least on the surface appear to be quite divergent. The purpose of this paper is to show that these problems are not divergent, that they have much in common, and that they are actually very much interrelated. In short, a single mathematical formulation will be developed that relates the two theories.

Before presenting such a combined formulation, a brief review is presented of the two basic techniques in very simple form. The purpose of this review is twofold—1. to re-establish the assumptions on which these two techniques are based, and 2. to establish a common set of symbols that will be helpful for the combined formulation.

A SIMPLIFIED BREAK-EVEN POINT MODEL

The historical development of break-even analysis and the break-even chart would be exceedingly difficult to trace. Like many other techniques, it was simultaneously created and developed by many people with the rapid growth of the scientific management movement during the early years of the twentieth century. It was one of the techniques that was quickly exploited.

Certainly Dr. Walter Rautenstrauch was one of the principal developers and proponents of this technique (7). It is to be found in practically all books which deal with engineering economy problems (1) (2) (3) (4) (10) (13).

Generally it is treated in its most simple form for solving short-range problems and the time value of money aspect of the problem is omitted. This is not always the case, and more sophisticated methods involving different basic assumptions have been developed. One of these is the MAPI formulation (6) (9).

Although many types of problems lend themselves to break-even analysis, a specific type of problem is of concern here—the economic comparison of two alternative methods of performing a task (actually any number of alternatives can be compared simultaneously by this procedure).

Costs for each alternative are divided into two categories and called either fixed costs or variable costs. The fixed costs are those costs that occur only once during the life of the alternative. Once these costs have been expended, they are not recoverable. Examples of such costs are:

1. Set up and tear down cost.
2. Cost of special tooling.
3. Paper work and clerical cost required to schedule a job.

In other words, the fixed costs are those costs that are not incurred by every unit of production. If we were to plot cost versus the number of units produced, these costs would appear as step functions. In the most simple problem, only those fixed costs that occur prior to the first unit of production are included. This is frequently done when the other fixed costs are assumed to be negligible.

Conversely, variable costs are those costs that are incurred by every unit of production. Examples of such costs are:

1. Direct labor cost per piece.
2. Direct material cost per piece.
3. Depreciation.

Once both types of costs have been identified for each alternative, it is possible to write a total cost equation for each one of the alternatives in terms of production quantity.

Total cost = (summation of all fixed costs) + (the production quantity) × (the summation of all variable costs)

Where: C_i = total cost of the ith alternative
f_{xi} = an initial cost of the ith alternative
F_i = summation of all initial fixed cost for the ith alternative

v_{xi} = a variable cost of the ith alternative
V_i = summation of all variable cost for the ith alternative
N = the quantity to be produced Eq. 1.

Then $C_i = (f_{1i} + f_{2i} + \cdots) + N(v_{1i} + v_{2i} + \cdots)$

Or $C_i = F_i + NV_i$

If C_i is plotted against N for each alternative, a simple graphical relationship results (Fig. 1). F_i becomes the y intercept and V_i becomes the slope of the straight line that this equation represents. If the two alternatives have been selected such that the one having the lower F_i has the higher V_i, then the two lines will intersect at some positive value of N. This point is called the break-even point. It represents the quantity at which the total costs are the same for both alternatives. When the demand for the product is greater than this break-even quantity, then the alternative with the lower V_i is more economical. When the required quantity is less than the break-even point, then the alternative with the lower F_i is the more economical.

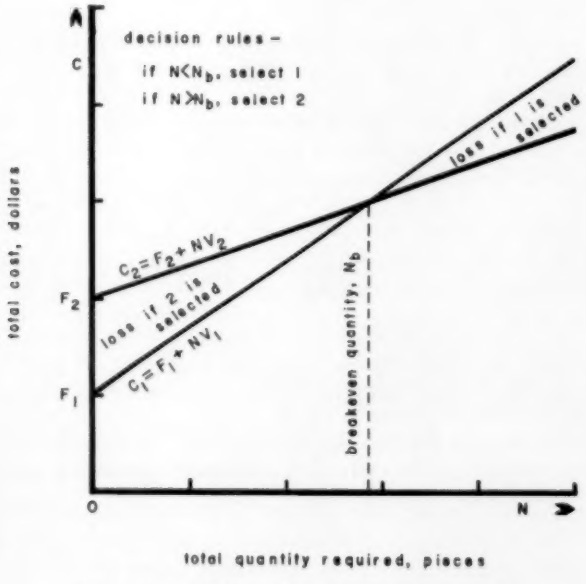

FIG. 1. A Break-Even Chart for Two Alternatives

In a quantitative manner this analysis tells us what we recognize intuitively—that a production method with relatively low fixed costs and high variable costs will be more economical at lower quantities while a production with relatively high fixed costs and low variable costs will be more economical at higher quantities.

It also reminds us that no single production method is best for all quantities of production. Since minor and major methods variation are easy to conceive through changes in machines, tools, men, motion patterns, etc., it should not be too difficult to imagine an almost infinite number of possible production methods from which to choose—each with its own small range of production quantities over which it is more economical than any other alternative (Fig. 2).

A SIMPLIFIED PRODUCTION ECONOMIC LOT-SIZE MODEL

Like break-even analysis, economic lot-size theory was developed during the period from 1910 through 1930. Its historical development has been clearly traced by Fairfield E. Raymond in his monumental treatise on the subject (8). Since this book tells the story so well and contains a complete bibliography, it would be superfluous to repeat it here. However, a table from this text showing the development of economic lot-size formulas has been reproduced here because it is so revealing.

TABLE 1

Equations Employed to Show the Stages in the Development of Formulae for Economic Lot Sizes

First appearance	Form	Authorities	Approx. date of record
1912	Cubic equation not published	G. D. Babcock	1912
1915	$Q = \sqrt{\frac{P \cdot S}{c} \cdot k}$	F. W. Harris	1915
		D. B. Carter	
		General Electric Co.	
		J. A. Bennie	1922
		P. E. Holden	1922
		K. W. Stillman	1923
		Benning and Littlefield	1924
		J. M. Christman	1925
		G. H. Mellen	1925
		Eli Lilly & Co.	1917
1917	Special adaptation		
1917	$Q = \sqrt{\frac{P \cdot S}{c \cdot i} \cdot k}$	S. A. Morse	1917
		W. E. Camp	1922
		Heltzer Cabot Co.	1924
		N. R. Richardson	1927
	$Q = \sqrt{\frac{P \cdot S \cdot k}{c \cdot (i + f_i)}}$ where f_i = allowances for insurance, storage costs, etc.	H. T. Stock	1923
		B. Cooper	1926
1918	$Q = \sqrt{\frac{P \cdot S \cdot D'}{c \cdot i (D' - S)} \cdot k}$	G. Pennington	1927
		E. T. Phillips	1927
		W. L. Jones	1929
		J. W. Hallock	1929
1918	$Q = -\frac{P}{c} + \sqrt{\frac{P^2}{c^2} + \frac{P \cdot S \cdot D'}{c \cdot i (D' - S)} \cdot k}$	E. W. Taft	1918
1923	$Q = \sqrt{\frac{P \cdot S \cdot k}{c \cdot i \cdot f_d + \frac{a \cdot b}{h} \cdot k' b}}$	G. Pennington	1927
		F. H. Thompson	1923
		R. C. Davis	1926
		C. N. Neklutin	1929
1924	$Q = \sqrt{\frac{P \cdot S \cdot k}{c \cdot i + (m + c) \cdot i_p \cdot S \cdot i}}$	A. C. Brungardt	1923
		P. N. Lehoczky	1927

The operations research movement stemming from the success of operations evaluations groups in the various services during World War II has given a rebirth to eco-

nomic lot-size theory. It is interesting to note, however, that the recent writings of many operations researchers seem oblivious of the work done during the 1920's. At least there appears to be some hesitancy towards acknowledging this earlier work (5) (11). Economic lot size as it relates to inventory control has also been treated recently

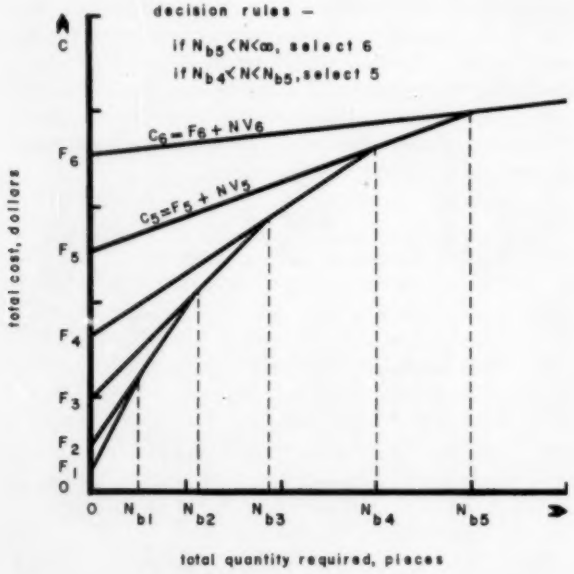

FIG. 2. Break-Even Chart for Multiple-Alternative Case

by T. M. Whitten in the theory of inventory management (12).

In the production of goods when the rate of production exceeds the rate of demand, the production must proceed in batches if the maximum production rate of the process is to be fully utilized. Otherwise, a ponderous growing stock pile will soon exist. The question arises as to the size of these batches or length of the run or the number of batches or lots to be started each year (in other words, four ways of stating the same problem).

In order to achieve the lot size that is most economical, two opposing cost elements must be considered. On the one hand, there are the costs that are incurred for each new lot, namely, set-up cost of the process including necessary scheduling, handling, paper work, etc. These costs are relatively proportional to the number of lots and would tend to make the lot size large. On the other hand, there are the costs incurred due to the storage of items necessitated by the production rate being higher than the demand rate. When long production runs are made, the amount of storage space required and the amount of money involved in inventory would increase. These costs would tend to make the lot size small.

Economic lot-size theory as does break-even analysis involves the development of the total cost equation for any specific situation of this kind.

Total cost = (the number of lots) × (the preparation and set-up cost) + (the production quantity) × (the sum of all variable costs) + (average inventory quantity) × (the inventory cost per unit during a given time period) × (number of time periods during the demand period) Eq. 2.*

Where C_i = total cost of the ith alternative
N = total quantity to be produced
n = lot size quantity
F_i = sum of preparation and set-up costs for the ith alternative
V_i = sum of the variable costs for the ith alternative
P_i = production rate (after set-up) costs for the ith alternative
D = the demand rate

$\frac{n}{2}(1-D/P_i)$ = the average inventory quantity (assuming a constant demand rate)

I = sum of storage and inventory costs per unit during a given time period

Then $$C_i = \left(\frac{N}{n}\right)F_i + NV_i + \left(\frac{n}{2}\right)\left(1-\frac{D}{P_i}\right)\left(\frac{N}{D}\right)I$$

Or $$C_i = N\left[\left(\frac{I}{2}\right)\left(\frac{1}{D}-\frac{1}{P_i}\right)n^1 + V_i n^0 + F_i n^{-1}\right]$$

If C_i is plotted against n for a specific alternative, a simple graphical relationship results for any specified value of D. This represents the graphical sum of the three costs in Eq. 2. (Refer to Fig. 3.)

By setting the first derivative of cost with respect to the lot size equal to zero, it is possible to find the lot size which yields minimum cost.

$$\frac{dC_i}{d^n} = 0 = N\left[\frac{I}{2}\left(\frac{1}{D}-\frac{1}{P_i}\right) - \frac{F_i}{n^2}\right] \qquad \text{Eq. 3.}$$

$$n = \sqrt{\frac{2F_i}{I\left(\frac{1}{D}-\frac{1}{P_i}\right)}} \qquad \text{Eq. 4.}$$

If this value of n is substituted back into Eq. 2., a total cost equation results which is the minimum cost for any value of N.

$$\min_{C_i} = N\left\{\left(\frac{I}{2}\right)\left(\frac{1}{D}-\frac{1}{P_i}\right)\sqrt{\frac{2F_i}{I\left(\frac{1}{D}-\frac{1}{P_i}\right)}}\right.$$

* This only is true where $P > D$.

$$+ V_i + F_i \sqrt{\frac{I\left(\frac{1}{D} - \frac{1}{P_i}\right)}{2F_i}} \Bigg\} \qquad \text{Eq. 5.}$$

$$\min_{c_i} = N \Bigg\{ \sqrt{\frac{I\left(\frac{1}{D} - \frac{1}{P_i}\right) F_i}{2}}$$

$$+ V_i + \sqrt{\frac{I\left(\frac{1}{D} - \frac{1}{P_i}\right) F_i}{2}} \Bigg\}$$

$$\min_{c_i} = N \sqrt{I\left(\frac{1}{D} - \frac{1}{P_i}\right) F_i} + NV_i$$

When this result is compared with Eq. 1., there is considerable similarity. The variable cost term is identical. The coefficient of the fixed cost term in the economic lot-size formulation is considerably more complex than that of the break-even formulation. In the break-even formulation, this coefficient is unity.

It should be apparent from the foregoing simple formulation of economic lot size that the concept of alternative manufacturing methods is totally ignored. Economic lot-size theory as developed here assumes one and only one manufacturing method. While this may be realistic enough after production methods have been established, it is not realistic at the time the manufacturing method is being established.

Thus two methods have been explored. Both of these methods purport to select the best method of doing a job. But these two techniques are based upon assumptions that are radically different. *Break-even analysis ignores inventory cost and assumes a single set up while economic lot-size theory ignores the possibility of different manufacturing methods with their accompanying differences and fixed and variable cost patterns.*

THE COMBINED MODEL

There are two possible ways of approaching combined formulation of break-even analysis and economic lot-size techniques. The simpler case is the discrete case which assumes a limited finite number of possible production alternatives. The more complex case is the continuous one which assumes an infinite number of alternative production methods. The first case selects the optimum method from several stated possibilities. The second case selects the cost pattern of the optimum alternative and then searches for the method which actually matches this cost pattern. Only the first case is discussed in this report.

To fully explore the discrete case, we will examine a problem involving three alternatives: namely, the age-old problem of whether it is better to use an engine lathe, a turret lathe, or an automatic lathe to perform a specified manufacturing operation.

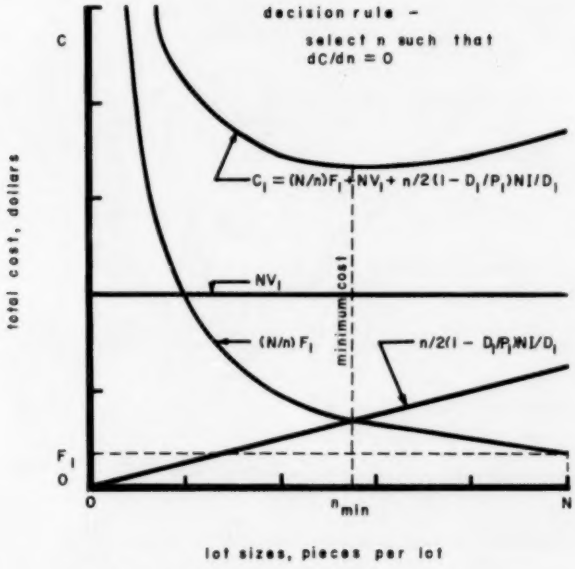

FIG. 3. Graphic Formulation of Economic Lot for A Single Alternative

To make this combined formulation as realistic as possible, the following values have been assumed:

1. Sum of costs that are incurred each time the job is set up (F_i).
   a. Engine lathe, $F_1 = \$1$
   b. Turret lathe, $F_2 = \$30$
   c. Automatic lathe, $F_3 = \$70$
2. Sum of all variable costs including direct labor, direct material, and manufacturing expense (V_i).
   a. Engine lathe, $V_1 = \$.20$ per unit
   b. Turret lathe, $V_2 = \$.10$ per unit
   c. Automatic lathe, $V_3 = \$.05$ per unit
3. Sum of all storage and inventory costs per unit of production when stored for *one month*. This amount is the same for all three alternatives (I).
   $I = \$.25$ per month per unit.
4. Production rate (P_i).
   a. Engine lathe, $P_1 = 5{,}000$ pieces per month
   b. Turret lathe, $P_2 = 15{,}000$ pieces per month
   c. Automatic lathe, $P_3 = 45{,}000$ pieces per month

All of these values are inherent in the particular production method and storage system. In a sense, these are fixed values. In addition, we must know or assume values relative to customer demand. We wish to examine what happens at different levels of customer demand.

To make this formulation more realistic the following levels have been assumed:

5. Demand rate (D).
   Case No. 1, $D_1 = 100$ pieces per month
   Case No. 2, $D_2 = 1{,}000$ pieces per month
   Case No. 3, $D_3 = 5{,}000$ pieces per month

Now it is possible to calculate the total cost equation for each alternative according to Eq. 5. This is done in the three steps shown as follows.

Step No. 1 (the basic equation)

Engine lathe

$$\min_{C_1} = N\sqrt{\frac{1}{4}\left(\frac{1}{D_j} - \frac{1}{5{,}000}\right)1} + .2N \qquad \text{Eq. 6.}$$

Turret lathe

$$\min_{C_2} = N\sqrt{\frac{1}{4}\left(\frac{1}{D_j} - \frac{1}{15{,}000}\right)30} + .1N \qquad \text{Eq. 7.}$$

Automatic lathe

$$\min_{C_3} = N\sqrt{\frac{1}{4}\left(\frac{1}{D_j} - \frac{1}{45{,}000}\right)70} + .05N \qquad \text{Eq. 8.}$$

Step No. 2 $\left(\text{let } K_{ji} = \sqrt{\frac{1}{4}\left(\frac{1}{D_j} - \frac{1}{P_i}\right)F_i}\right.$ and find the K value for each alternative at each demand rate—9 cases in all.)

	D_1	D_2	D_3
K_1	.049	.014	0
K_2	.274	.084	.031
K_3	.42	.132	.056

Step No. 3 (substitute K values into Eq. 6., 7. and 8. and combine terms).

	D_1	D_2	D_3
$\min_{C_1}$	$.249N$	$.214N$	$.2N$
$\min_{C_2}$	$.374N$	$.184N$	$.131N$
$\min_{C_3}$	$.425N$	$.182N$	$.106N$

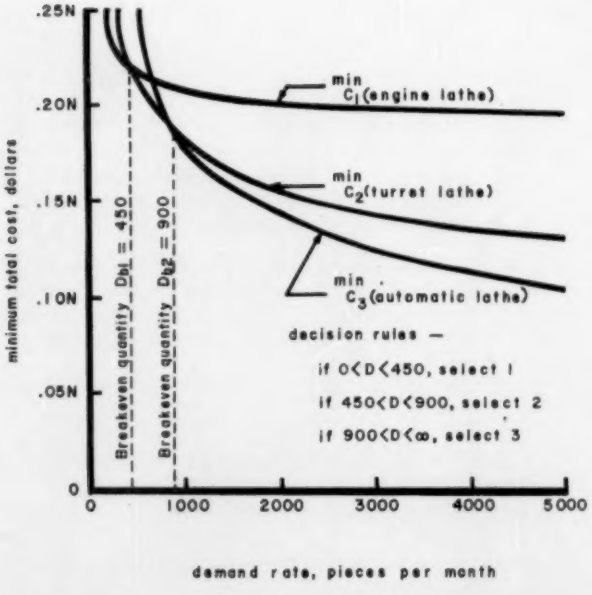

FIG. 4. Combined Formulation for Three Alternatives

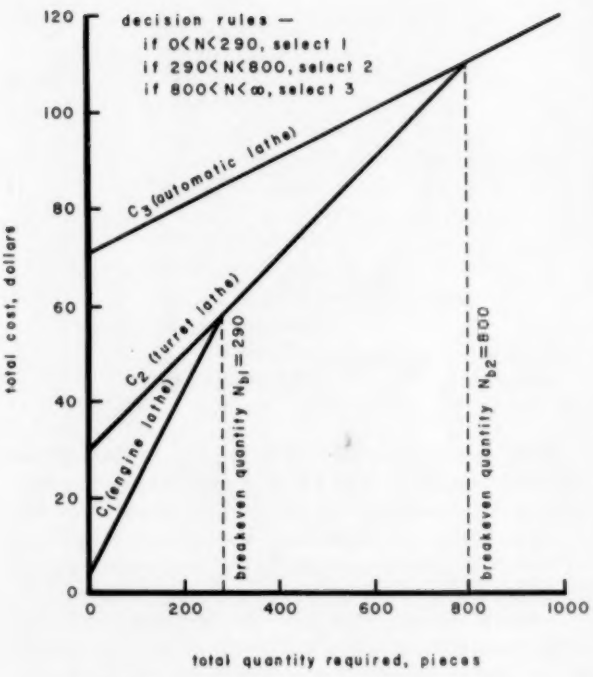

FIG. 5. Break-Even Chart for Three Alternatives

It is now possible to plot the total minimum cost of each alternative against the demand rate. Figure 4 shows this relationship.

When a smooth curve is drawn through the points for each alternative, three break-even points become apparent. The engine lathe is the most economical method when the demand rate is greater than 0 and less than 450 pieces per month. The turret lathe is the most economical when the demand rate is greater than 450 pieces per month and less than 900 pieces per month. The automatic lathe is the most economical when the demand rate is greater than 900 pieces per month.

How do these answers compare with those obtained from the typical break-even analysis? In the first place, it should be pointed out that these answers are entirely independent of the total quantity required (N). Thus the answer obtained is much more general. To compare these answers with the standard break-even analysis, it is necessary to assume various values of N.

Substituting known values into equation 1 yields the following total cost equation for each alternative according to the break-even analysis model which has been developed previously:

Engine lathe, $C_1 = 1 + .2N$ Eq. 9.

Turret lathe, $C_2 = 30 + .1N$ Eq. 10.

Automatic lathe, $C_3 = 70 + .05N$ Eq. 11.

When the C_i versus N curves are plotted, three break-even points are once again apparent. The engine lathe is

the most economical method when the quantity required is greater than 0 and less than 290 pieces. The turret lathe is the most economical method when the quantity required is greater than 290 pieces and less than 800 pieces. The automatic lathe is the most economical when the quantity required is in excess of 800 pieces. This relationship is shown in Figure 5.

COMPARISON OF RESULTS

The next problem is to find a convenient and graphic way of portraying the errors that would have been generated in this specific problem if the simple break-even method had been used to solve the problem. Or to state it another way, what savings would have been derived by using the more complicated, combined model in determining the best production method?

A glance at the decision rules which are established in this example by the break-even model and the combined model is revealing because in most situations the two methods result in different courses of action rather than the same course of action. Figure 6 clearly reveals the disparity between the two methods. Listed below are the types of errors that can be made. These are shown graphically in Figure 6. In each case the area over which the error can be made is a rough approximation of the probability of this type of error. The actual loss at a specified point in the Figure 6 matrix may be determined from Figure 4.

TYPES OF ERROR

Type of Error	Breakeven Model Chooses	Combined Model Chooses
1	Engine Lathe	Turret Lathe
2	Engine Lathe	Automatic Lathe
3	Turret Lathe	Engine Lathe
4	Turret Lathe	Automatic Lathe
5	Automatic Lathe	Engine Lathe
6	Automatic Lathe	Turret Lathe

Generally a sales forecast would be able to limit to some degree the space in this figure that is actually pertinent. Let us say in this example that we are reasonably certain that the total volume required will be between 1200 and 2100 pieces and that the demand rate will be between 200 and 700 pieces per month. If this were the case checking Figure 6, it is apparent that the break-even model would give us the wrong decision over this entire space. According to the break-even model, we should always use the automatic lathe. According to the combined model, we should either use the engine lathe or the turret lathe depending on the point in this space that actually occurs.

CONCLUSIONS

1. The use of the simple break-even model to decide between several alternative methods of performing a task can give rise to serious errors.

2. These errors are particularly pronounced when—

   a. The production rates are considerably higher than the demand rates.

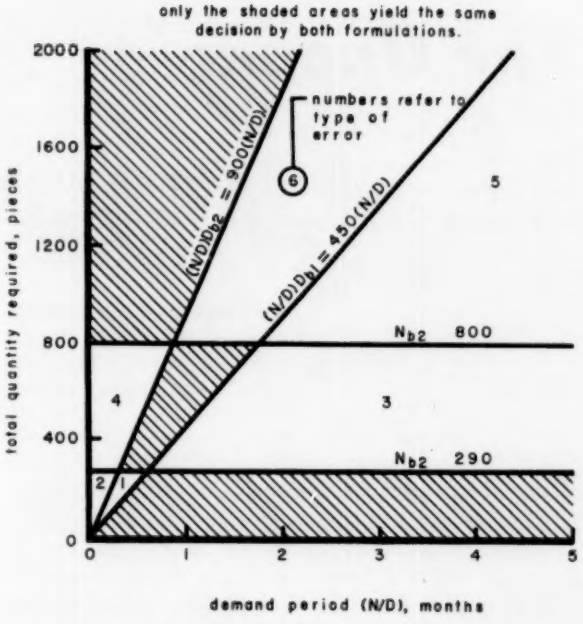

FIG. 6. Error Space Diagram

   b. The storage and inventory costs are significantly large.

   c. There is a substantial difference between the ratio of set-up cost and storage cost for the several alternative methods.

3. The combined model is not significantly more difficult to work with when modern computing techniques and equipment are considered.

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The Unopar

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Assistant Professor
Department of Industrial Engineering
Washington University

THE UNOPAR is the first device to ever measure the actual structure or anatomy of work. Now it is possible to learn what actually occurs in the physical performance of a job.

Work is the central core of all Industrial Engineering functions. Many definitions of work have been provided. Do we really know what work is? In most aspects, our knowledge is still inadequate. If Industrial Engineers are to deal effectively with work, more information must become available to cope with both the new problems of automation and non-repetitive activities, as well as our usual problems.

How can we approach the concept of work? From a practical point of view, it is necessary to investigate the factors related to work. The easiest approach for this discussion is a cause and effect subdivision of these factors:

1. Factors actually having an effect on the work, or causing an effect on the work (causal).
2. How these factors finally exhibit themselves during the work performance (effect).

From a superficial point of view, it would appear that it would be best to study the factors affecting work. These causal factors in human beings and the work situation are difficult to study. Since many investigations have been performed to learn about these causal factors, a very brief look into the types of activity would help clarify what is meant by causal factors.

RESEARCH INTO CAUSAL FACTORS

In the physiological area heart rate has been measured in attempts to find a relationship with work (11). H. H. Young has performed work which is directed toward finding a relationship between heart rate and operator pace. Brouha has done both heart rate measurements and energy expenditure measurements in his laboratory.[1] A new device, the Lauru Platform, has been employed to measure energy expenditure (5). Energy expenditure experimentation with metabolism measurements even went so far as to lead to an attempt to develop an a priori procedure for measuring energy (10). Even the work of some of the earliest physiologists like Atzler and Fenn could be classified in the area of trying to learn about factors affecting work.

Sociological and motivational research is being performed more frequently now. One of the most famous sociological attempts at finding correlates between output of work and other factors was the Hawthorne study. Today there are several activities, like the University of Michigan surveys, which attempt to find some relationship between group feelings or other factors concerning work, and work output.

Psychological research has been performed for many years. Research has been conducted in many areas, like personal external factors, temperatures, noise, and ventilation. Psychology is continuing its research to help find some factors which have relationships with work output.

All research into causal factors is done basically for two purposes: 1. to gain an insight into work, and 2. to establish correlations with the output characteristics (time or production) for the purpose of analysis and design of work (methods and conditions).

It is quite apparent that the first purpose has been well met. The insights which we have into work have been forwarded.

Progress in the second purpose is not significant because of the tremendous complexity in the interaction of factors. In addition, the output characteristics used in this type of investigation, have been gross, like time per unit or number of units per time interval. It has not been possible to measure accurately any part of the structure of the work performance itself. This apparently is a large drawback to developing effective conclusions. Certainly, it should be expected that some day through the diligent research approach these factors affecting work and their complexities will be isolated and related. But this is many years away.

STUDYING THE EFFECTS OF CAUSAL FACTORS

This in a sense indicates that it is easier, and there is greater likelihood of productive results, to study the *effects* of these factors for the same two purposes, that is, gain an insight into work, and find some correlation between these effects and output characteristics.

To become more specific at this point, it is necessary to reduce the scope of this presentation of work to that which is important to the work simplification and measurement area. This does not mean that work chararacteristics and their correlates are unimportant in the other activities of Industrial Engineering. Rather, the area which can make immediate use of such information is work simplification and measurement.

One of the main areas where the measurement of work and its relationship to output characteristics are impor-

[1] Haskell Laboratories, duPont Co., Wilmington, Delaware.

tant is the "rating" concept in time study. Each present rating concept is really an attempt at measuring the effects on the human being of all factors affecting the work. The Westinghouse or levelling system was one of the first real attempts to define factors affecting work (6). The skill and effort ratings were the "measurements." At this time, it is simple to show that such measurements are inaccurate and are difficult to make because of their dependence on human judgment.

Then speed rating became the next general attempt at measuring the effects of all factors. This proposal for speed rating came basically from Barnes and Carroll (1) (2). The next proposal was one of attempting to measure by human judgment the acceleration of human motions (7) (9). The mere fact, that human judgment is needed to measure speed or acceleration, points out the difficulty of obtaining accuracy with these concepts.

These proposals did provide some insight into work for Industrial Engineering, and were developed as a result of other activities which gave some insight to these men. To be useful, the effects of the casual factors must be measurable. Without much discussion, it would be simple to show that we cannot today measure speed, skill or effort (9). The suggestion that acceleration be measured left much to be desired, because, for one reason, present techniques could not provide even a rough approximation of this value.

WHAT IS TO BE MEASURED?

In effect, these conclusions about the inability to make these measurements require a review of the definition of work which says it is the motion of a point of application of a force. Usual engineering applications of this principle have concerned the measurement of the *force* since the other parts of the expression are usually known. However, it becomes apparent that in Industrial Engineering the biggest factor about which we know little is the *motion* from which it would be possible to determine amount of work. Therefore, it is essential to measure, in all its variations, the motions performed by people because 1. it is the important unknown factor in the engineering definition of work and in all Industrial Engineering applications, and 2. it is the final effect of all causal factors affecting work.

Now, what constitutes the measurement of a motion? Fortunately, this question has been well answered by other branches of science. What must be measured is the velocity, acceleration, deceleration, position in space, distance and time of a motion. It had not been possible to measure these factors for human motions until the UNOPAR (Universal Operator Performance Analyzer and Recorder) was developed. To date, research and industrial techniques have determined only time between two points for a motion (time study, Kymograph, etc.), or a rough approximation of the motion path (motion pictures, chronocyclegraph, etc.). Virtually nothing has been learned about what happens between the points, or how to translate a rough motion path presentation into the measurements needed for a motion. These techniques, and others in the areas of psychological and physiological research, have been useful to date because they have helped in learning about work. They will be used in the future for providing the type of information they give best. But their use will be assisted greatly by the information UNOPAR can provide, and their data can be correlated with UNOPAR data to provide even wider uses for these older techniques.

With these essential measurements made accurately it will be possible to measure most aspects of work, and to find what is the true effect(s) of all the factors affecting the work.

WHAT UNOPAR DOES

The UNOPAR was developed to measure the foregoing items because they are the true final effects of any and all factors affecting the work. These measurements are the means whereby it will be possible to describe in complete detail the exact method used on a job. Obviously, the "exact method used" is the final effect of all factors. It is work.

DEVELOPMENT OF UNOPAR

In designing a new device for making these measurements, certain factors had to be considered. The measurements must be three dimensional. Any attachments to the body member studied must be small and light. Any device developed must be capable of being used under actual factor conditions. The range of velocities might be rather small (0 to 10 feet per second). The range of acceleration and deceleration could be quite large and so forth. By exploring the application of electronics in this problem, a rather suitable solution, the UNOPAR, has been obtained (3).

Initially, there were a number of possible measuring techniques which could conceivably fulfill some or a great number of the aforementioned requirements. After a careful consideration of all these possible measuring techniques, it was decided that the Doppler effect with sound as the radiation medium presented the fewest formidable obstacles for successful development of a work measuring device.

FUNDAMENTALS OF THE UNOPAR

The operating frequency of the UNOPAR was chosen at 20,000 cycles per second which is above the threshold of hearing at normal levels of intensity. This frequency helps overcome the interference problem from other noises within the factory or laboratory and would not disturb the performance of the operator. The sound emitting source (speaker) must be attached to the body member (wrist, finger, elbow, etc.) which is to be studied; however the size of this source is smaller in diameter, thinner and lighter than an ordinary wrist-watch.

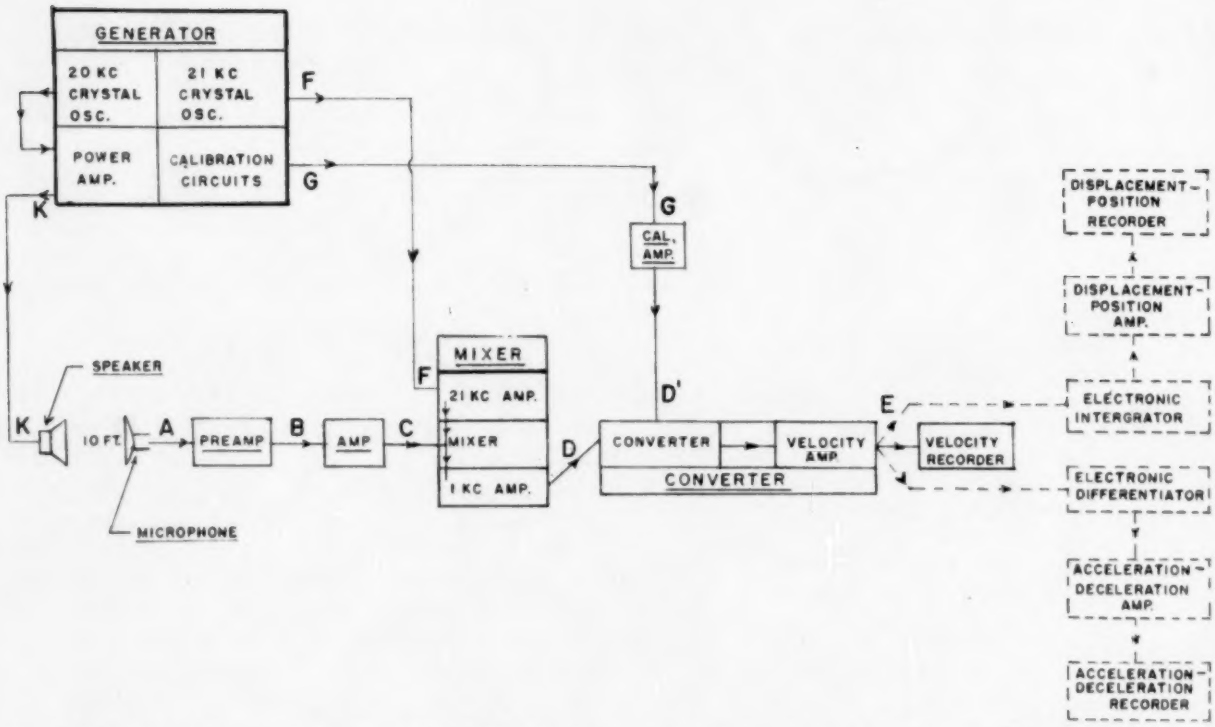

FIG. 1. Block diagram for a single channel of UNOPAR. The other channels which are used for measurements of two and three dimensional motions are identical. Not shown on this diagram are the computer components used to combine the three vectors of velocity, the three of acceleration-deceleration and the three of displacement-position to give the resultant values of each for a three dimensional motion.

Three microphones, oriented in three planes, with the directional axis of each perpendicular to the directional axes of the others, are utilized to measure motions in the three planes. Each microphone receives its component portion of the sound waves emitted from the speaker attached to the body member moving towards or away from the microphone. This system provides a variable reference orientation, since the three planes can be rotated in any position as long as the orientation with respect to each other microphone is fixed. This flexibility makes it possible to circumvent obstacles to the path of transmission and to permit use of overhead space in industrial situations.

Because of the Doppler effect, a transmitted source of sound (speaker) that is in motion relative to a stationary receiver (microphone) provides an increase or decrease in frequency of sound waves at the receiver which varies directly with the velocity of the source. Motions toward the microphone will increase the frequency received, and motions away will decrease the frequency received.

To minimize the error which can be caused by the moving speaker not remaining in the direct axis of each stationary microphone, the motion of the speaker is assumed to be confined within a one yard cube and the microphones are placed ten feet away from the center of this cube. The maximum possible error of measuring the velocity and time (the two basic measurements made with UNOPAR; the other measurements are made by mathematically and electronically treating the two basic measures) will not exceed 1% of true value. Because 1. almost all work activity is usually performed with a one yard cube, 2. the size of the cube and the distance of the microphones from the cube can be increased (with the appropriate adjustments in UNOPAR), and 3. it is possible to orient this cube in any manner to encompass most of the motions in an operation, this error problem becomes virtually non-existent for practical purposes.

OPERATION OF THE UNOPAR

The basic operating procedure of the UNOPAR begins with the generation of the operating frequency of 20,000 cycles per second (by crystal controlled oscillator at 20,000 KC.). As is indicated in the block diagram (Figure 1), this 20 KC. signal is sent through a power amplifier to the speaker. Most of the research work performed to date has been done by moving a speaker rather than by attaching a speaker to a moving body member. This has helped us learn more about the potentialities of UNOPAR. In the near future actual body motion tests will be performed. If more than one body member is to be studied, a different frequency signal would be used for each member involved.

At the present time the Lorenz electrostatic speaker, Figure 2, is being utilized in the experimental work. Present speaker developments indicate the actual size of the speaker will be reduced to the size of a thick nickel or quarter. This size will make it possible to study any type of physical work, because the speaker will not hinder the operator in any way.

The perpendicular vectors of the frequency changes are received by the three microphones. The block diagram (Figure 1) shows only one microphone and its corresponding circuit; the same circuitry would be used for the other two microphones. When the speaker is in motion, the frequency of the sound received at the microphone will be 20 KC. per second plus or minus the Doppler difference caused by the motion of the speaker. Since the voltage level of the received signal is low (175 micro-volts), it is sent through a preamplifier attached to the microphone stand. From here, the signal is sent to an amplifier section of the UNOPAR electronic relay rack. This amplifier has seven tuned amplifier stages.

The 20 KC. signal, plus or minus the Doppler difference frequency, is mixed with a 21 KC. reference signal (generated by a crystal controlled oscillator of 21,000 KC. control). From this mixing, a 1 KC. signal, plus or minus the Doppler difference frequency, is obtained. This is the usable signal for conversion to voltage. It is sent through two tuned amplifier stages to the converter chassis.

In the converter, the frequency sine wave is first changed to a square wave. This makes the signal which drives the converter (2 D21 Thyratron) independent of the amplitude of the incoming signal and dependent strictly on frequency change. In addition, the thyratron must be driven by a signal (pip) of high amplitude and short time duration (obtained by differentiating the square wave; this could not be obtained directly from the sine). The pips are sent into the thyratron whose circuit is arranged to provide a D.C. voltage output directly proportional to the frequency of the input pips. Since the sum of the input pulses remain on the output voltage, the signal is sent through a pie section filter. The resultant D.C. voltage is directly proportional to the velocity of the speaker.

The D.C. voltage for each plane of motion is sent through an amplifier to the three velocity recorders. Each amplified D.C. voltage is also sent to an electronic differentiator, whose output voltage is proportional to the deceleration and acceleration of the speaker. Each of these signals is sent to an acceleration-deceleration recorder. Each amplified "velocity" voltage is also sent to and electronic integrator whose output voltage is proportional to the displacement of the speaker. Each of these signals is sent to a displacement position recorder. Direction of the motion is determined from the voltage variations above or below the voltage output of the one KC. signal input. A larger voltage indicates a motion toward the microphone, and a smaller voltage indicates a motion away from the microphone. The three records for a given measurement unit can be combined into an absolute total by electronic vector summation.

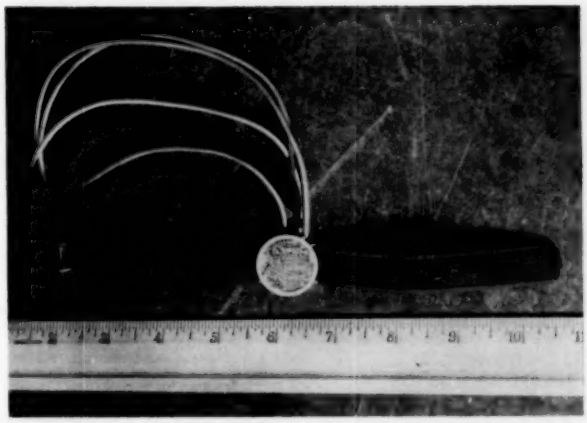

FIG. 2. The new specially designed Lorenz Electrostatic Speaker is only 1" in diameter, ¼" in thickness and weighs less than one ounce. It is shown with a regular watchband and the two small wires used to operate it.

The calibration circuit is designed to generate sine wave signals of the proper frequency and voltage to check the velocity and acceleration maximum displacement values, and the zero set of all the electronic equipment.

A 12 channel D.C. oscillographic recorder (Figure 3) is used to plot the individual vectors of velocity, acceleration-deceleration, and displacement position. The summation vectors of velocity, acceleration-deceleration, and displacement are also plotted on this recorder. The recorder is capable of plotting information at a constant synchronously controlled maximum speed of 125 mm. per second (5 inches per second), thus providing an accurate timing record for every motion.

FUTURE DEVELOPMENTS OF THE UNOPAR EQUIPMENT

The present electronic equipment and recorders are fairly large. Development of the equipment indicates there will be a substantial reduction in the amount, size and weight of the necessary components. At the present time all the electronic equipment must be in operation when measurements are to be made. However, in the near future a 3 channel magnetic tape-recorder will be introduced into the circuit, between the mixer amplifier and the converter chassis. One channel of the tape-recorder will be utilized for each one of the three planes. The inclusion of the tape-recorder makes UNOPAR a much more versatile measuring device, since only a small part of the electronic equipment need be present at the operation when measurements are being made. The tapes can be taken back to an office or laboratory and analyzed at a later time at the leisure of the analyst. Magnetic tapes

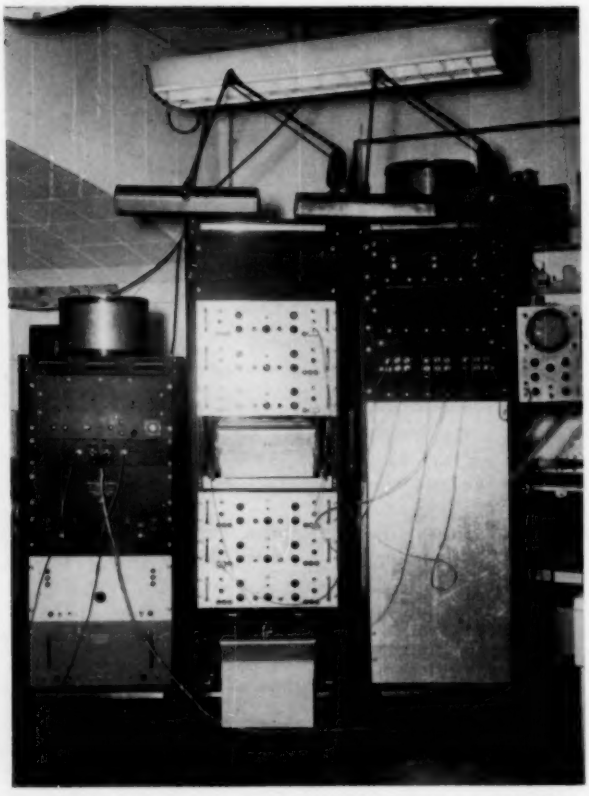

FIG. 3. The 12 channel brush oscillographic recorder unit is shown between the UNOPAR equipment racks. Inter-rack connections have been made to illustrate typical equipment set up for studying three dimensional motions.

provide a simplified and compact system of storing information, a much more practical way of utilizing UNOPAR, and an easy method for referring to any motion for new measurements or repetitions of the measurements.

Another problem to be solved concerns the handling, processing, and interpreting the great mass of data provided by the UNOPAR. At the present time, all the information is plotted in graphical oscillographic form. Each motion made by the operator provides data of velocity, acceleration, deceleration, displacement, position, and time. A digital computer would greatly facilitate handling and interpreting these data and performing any mathematical manipulations desired. Since the information from UNOPAR is in analog form (a continuous plot as every motion occurs), a conversion has to be made.

At the present time, the plotted curves are measured visually using a specially developed UNOPAR tape analyzer, and these measurements are punched into a digital computer input card. This requires a great deal of time and effort. To reduce greatly this additional work, the output of the UNOPAR can be fed directly into the analog-to-digital electronic converter. The converter takes the form of punched cards of magnetic tape suitable for input purposes to the digital computer. This proposed system for data handling is now in the preparation stage.

RELIABILITY OF UNOPAR MEASUREMENTS

Tests for the reliability of UNOPAR measurements were made for a one plane motion (there is no criterion for a three plane motion) with an accelerometer as the criterion source. The test was made in an anechoic chamber. This was done to eliminate all extraneous noises and superfluous reflections which might interfere with the accuracy test measurements. The speaker and accelerometer were rigidly mounted on a frame for the one plane motion.

Chi-square and correlation techniques were used to check the size and shape of UNOPAR velocity and acceleration-deceleration plots with the similar plots from the criterion accelerometer. Chi-square probabilities of 0.99 and above and correlations of 0.99 and above were obtained, indicating excellent fits for the two sets of curves. Additional verifications were made by calculating the standard deviation for percentage of error from the accelerometer plots for the maximum (peak) velocities, maximum acceleration-deceleration values, and time for a series of motions. For velocity, the standard deviation was 1.08 percent; for acceleration-deceleration, 2.17 percent; for time for velocity, 1.06 percent; and for time from acceleration, 2.79 percent. These results indicate excellent measurements from UNOPAR (there is evidence that indicates part or most of the errors obtained were due to the mass action of the accelerometer, not to UNOPAR).

A sample of the type of information available from UNOPAR is shown in Figure 4. The velocity plot in Figure 4 shows the direct response of the device to a motion of this type. This was a critical test of the transient response of the system since the speaker was brought to zero velocity almost instantaneously. Another illustration of a velocity curve from UNOPAR is shown in Figure 5. This plot is from a research project currently in progress.

USES OF THE UNOPAR

Because the UNOPAR provides the measurements of actual operator performance, many uses and benefits can be expected in the Industrial Engineering and motion and time study fields. Other disciplines which make studies of human performances will be able to use such a device. A brief review of some of the problems UNOPAR can help solve and the possible uses of the UNOPAR is given (3) (4) (8) (9).

Notice how insight into work is obtained, and correlates with output characteristics should be more readily established.

1. What is a motion? No one really knows how a position, or grasp, or even a transport loaded is performed. From UNOPAR measurements, these questions will be more readily answered. This type of information can eventually result in an adequate and correct list of therbligs and subclassifications.

2. What effect does one motion have on another? Since motions are always performed as part of a series, a catalog of the effects of one motion on another is a valuable possibility. And even with a catalog, the UNOPAR can always be used to check any specific operation.

3. Which method is best? Evaluating methods for an operation is at best a rough estimating procedure today. With UNOPAR information, method differences can be evaluated accurately to aid in selecting the best of the available methods or proposals.

4. Evaluating differences among operators on the same job. Why is there a wide difference in production between the best and poorest operator on a job? What does the good operator do that the poor operator doesn't do? Studying the motions with UNOPAR will assist in transferring good practice to others on the same job.

5. Making a methods description for work measurement. With the complete information about motions available, a complete and accurate methods description becomes possible. In addition, the selection of "proper" gradations in motion performance can be made, because there is likely to be a slight difference in motion performance from one operator to another. An extremely likely possibility is quantitative methods descriptions, which would give the accepted motions with allowable ranges in terms of velocity, acceleration or deceleration to indicate what will be considered the "same" methods.

6. When has a method changed? Because a standard or allowed time must be associated with given methods, many industrial problems arise through the decision to "change a standard because the method has changed." The worst problems are associated with minor method changes. With the UNOPAR records (and eventually with quantitative method descriptions) making this decision becomes much more objective.

7. Accurate timing for time studies. UNOPAR at present can measure time in units of 0.000133 minute. It can measure with less accuracy (bigger time units) if desired, and it could probably be made to measure smaller time units. Better yet is the fact that UNOPAR times are not human-interpreted time records as are obtained with the ordinary time measuring techniques (like stopwatch and motion pictures).

8. Rating or measuring operator performance. This is the biggest problem in time study and even in many phases of labor-management relationships. Several proposals (like speed and acceleration) have been made to solve this problem, but there has been no accurate measurement available for these factors. Now UNOPAR will be able to determine what it is that really affects work so that accurate measurements will always be made. After labor and management in a company select the standard amount of these units (that are highly related to work output), there should be no difficulty in almost all operator performance measurement. It will also be possible to compare performances on all jobs in all plants in the country with any other jobs through UNOPAR measurements, even though the standard units selected for a specific plant are different from those in another plant.

9. Improving other rating procedures. Just because UNOPAR is and will be able to make so many advances does not mean other Industrial Engineering tools will no longer be needed. UNOPAR will do many new things and it will greatly assist in making present techniques even better. Rating is one of these techniques. Ratings using human judgment will always be needed for long cycle or short run operations. Such ratings will be made more accurately and consistently than at present when rating practice and training is done with operations or films of operations where correct ratings or proportionalities of operator performance are obtained from UNOPAR records. The same approach can be used for many other techniques, some of which are mentioned in this article.

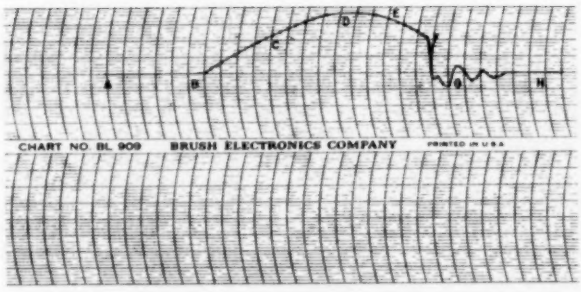

FIG. 4. A section of the UNOPAR velocity vs. time tape on which is recorded the one plane motion of the original test speaker. In this motion the hand moves the speaker from rest, over a known distance and strikes the front end brace of the test apparatus to check the transient response of the system. The straight line section of the tape between points A and B shows zero velocity (center line of chart is zero) indicating the hand at rest. As the hand began to move the speaker down the rod, the velocity charted increased rapidly from point B to C to D. The changing velocity along this section of the curve (slope) is the acceleration portion of the motion. At point D, the velocity reaches its peak. The curve is not flat (does not remain at a constant velocity) on top but changes smoothly from its rising characteristic (acceleration) to a falling one (deceleration) as the hand begins to slow down (D, E, and F) before striking the end support. The speaker struck the barrier at point F and the velocity dropped immediately. Slight rebounds (damping) of the hand and speaker frame are indicated by the wavy motions of the curve at Point G. The straight line at Point H indicates that the hand had come to rest again. This motion was recorded with the tape running at the fastest speed of 125 m.m./sec. Therefore, each large division between the curvilinear vertical lines on the tape represents a time interval of 0.000665 minute. The time interval for the performance of this motion is easily computed.

10. What is the effect of mechanical factors on operator performance? Ordinarily, mechanical factors are considered job difficulty factors, like weight, eye-hand coordination, foot pedals, and amount of body used. These factors reduce the pace at which an operator can perform. But how much? And in what relationship? Getting these answers will assist methods design as well as work measurement.

11. What are correct allowances? Many allowance categories, like personal time, machine time, and even irregular occurrences, depend on operator performance in some way. As pointed out above, wherever operator performance has a bearing on work, UNOPAR can assist. Again, UNOPAR will develop general data for all jobs, and specific information for an operation needing close study.

12. What is fatigue? This subject is thoroughly discussed in the literature, yet so little is known about it. There is no question that it is highly related to work, and learning about how a person works through UNOPAR records will provide insight into those factors which might be considered related to fatigue even more than our present techniques permit us to do. Some procedures for compensation, if needed, for fatigue, in terms of both time standards and wage standards, should evolve from this information.

13. Are predetermined motion-time systems accurate and consistent? Although these systems do have an important use in Industrial Engineering this question has never been settled,

mainly because so much depends on finding answers to the other questions posed above: How are motions performed? What is the interaction of motions? What is the exact method being used on a job? How to rate accurately, and many others. Because UNOPAR will help find these answers, it can likewise be expected to shed a great deal of light on this highly debated issue.

14. Selection of people for jobs. Present selection tests (using a work situation) all use some output characteristic (like time for assembling pegs in a board) as a measurement for guiding selection. Here, like in work simplification and measurement, no information is obtained concerning *how* the work was done. For example, it might be better to select a person for the ability to perform a series of motions within specified limits of velocity, acceleration-deceleration, and position than on the basis of time only. Such information could indicate a better ability to learn and make progress than could time alone.

15. Training operators for jobs. Once a motion pattern in terms of UNOPAR information is known (using skilled operators to provide the pattern), a training program can be developed easily to tell operators what is to be done. For example, the trainee could be told to accelerate rapidly on motion A, but slowly on motion B. Or to achieve a certain velocity in assembling. UNOPAR records could also serve as guide posts to the trainee, by comparing his record at regular intervals of time to the "desired" record. Of course, each person will vary in his own performance compared with other operators, but this training procedure starts an operator in the right direction.

16. Labor-Management disputes on methods and time standards. Information from all the above problems where UNOPAR will be used will do much to eliminate disputes concerning work methods and conditions, and incentives. But UNOPAR records of methods at, say the time of an original study, will be of great help in establishing exact conditions in the past, and in providing an excellent basis for comparison with the present situation.

17. Other Industrial Engineering activities. Since all of Industrial Engineering starts or deals with work, UNOPAR should help a great deal. For example, materials handling depends on people to move material, drive trucks, operate hoists and push dollies. Production control requires information on scheduling, dependent on time standards. Inventory control depends on storage methods—heights, weights, distances. Incentive systems depend on knowledge of what should be expected (standard from a work situation), and what might be expected (incentive level). Job evaluation requires information on skills, effort, dexterity and other items. Even the techniques of operations research frequently depend on adequate knowledge of human performance. Automation will depend on information concerning man's abilities and capabilities under non-repetitive work situations. With UNOPAR delving into the fundamental structure of work, all Industrial Engineering techniques will benefit.

18. Other branches of science. Measurements of human performance are needed in other areas of science.

(*a*) Physiology requires information regarding the external features of performing different tasks for correlation with physiological phenomena.

(*b*) Psychology is interested in the reaction of operator's motions and motion paths as related to certain psychological stimuli.

(*c*) The field of biomechanics is interested in the relationship between man and machine, and much of the work depends on information concerning what an operator does and the time or pace for the performance.

(*d*) The human engineer is interested in the relationship between man and machine in the practical situation. He realizes modern equipment usage usually relies, on the ability of an individual to learn a motion pattern rapidly, and then to maintain that pattern under almost all operating conditions. A complete understanding of this relationship will aid in the design of equipment and location of controls for the most effective utilization by the human operator.

(*e*) Physical education is directly concerned with motions, motion paths and learning more about what an athlete does in his performance. For example, an excellent golfer's motions might be analyzed with UNOPAR to assist in training, similar to the procedure previously outlined.

(*f*) The effects of mental illness on the performance (motion pattern) of a job could help find where such persons might be profitably utilized. UNOPAR records from these people could conceivably be used as aids in determining when a person might have some type of mental disease.

(*g*) Physically handicapped persons could also be studied to find information about their usefulness. Such data would assist materially in designing work methods for this type of person.

(*h*) Military possibilities for the use of UNOPAR are tremendous. What is the proper load and distribution of load for a soldier charging across a field? What does a jet-plane pilot do in operating his controls? Obviously, there are literally hundreds of such problems.

UNOPAR TODAY

These advantages to be gained through the use of UNOPAR are the direct result of the ability to obtain exact information about *each* and *every* motion of *every cycle*. Other procedures only obtain gross overall information. The ability to provide specific measurement information is a basic requirement for any professional endeavor.

At the UNOPAR Laboratory of Washington University, a research project concerning the study of the effects of learning on the performance of the motions of a particular job is being undertaken. This research is being carried out under contract with the Office of Naval Research. Figure 5 is a sample of the UNOPAR results for this experiment. There are significant indications of the learning effect which can be readily observed from the UNOPAR tapes themselves. A more complete and detailed disclosure of these and other effects will be reported at a later date when all the information has been processed, tabulated and statistically analyzed. Reports of all research with UNOPAR as the major measurement instrument will be made available as they are prepared.

Although so much information about work can be obtained with the UNOPAR, it is not ready at present for industrial application. Much more work must be done before proper interpretation of the measurements can be made. The problems in this respect are many fold. Should each motion be considered a separate element? Are individual motions truly separable? How should acceleration, deceleration, velocity, etc., be combined or related to obtain "an average" for regular time study elements? How are the records to be interpreted for use of these criteria in combination motions? Are these criteria correct indicators of operator performance? Once these answers are available, industrial measurements will be meaningful and important.

Fortunately, the UNOPAR itself will help solve these problems. For example, if acceleration-deceleration is not the correct measure of operator performance, the UNOPAR will make available the data to decide what the measure of performance should be. Notice that all these problems can be more easily solved for the first time in Industrial Engineering, because *actual measurements of every aspect of every motion as it occurs in the work performance are obtained.*

UNOPAR has not been fully developed. However, it represents such a radical change in the concept of measurement of human performance that there is every possibility it will revolutionize many aspects of Industrial Engineering as well as related sciences.

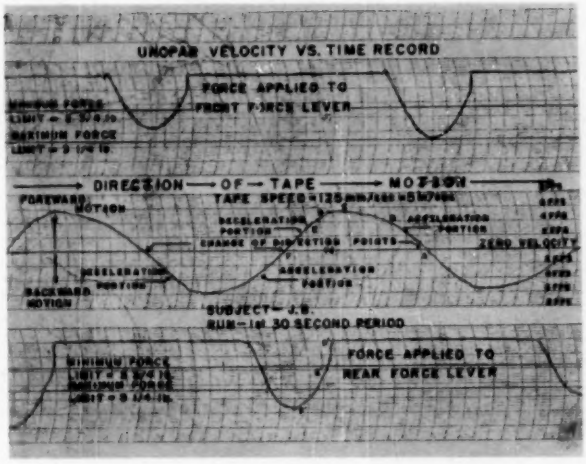

FIG. 5. A section of the UNOPAR tape record taken from a current research project being performed for the Office of Naval Research. The UNOPAR velocity vs. time record is shown on channel 2 (center) of this tape and a force measurement is shown on channels 1 and 3. This research project was undertaken to study the effects of learning on the performance of a job. The job was one of learning to apply a certain force (within given tolerance limits) at both ends of a 24" motion. A résumé of a typical cycle shows that the motion starts (point A) at the front end of the test table after a force has been applied to a force lever at this end. The subject moves with a rapidly increasing velocity from point A to B and then to the peak velocity at point C where he begins to slow down in anticipation of striking the rear force lever. The jog in the curve (sharp deceleration) at point D indicates that contact with the force lever has been made (see also point D') and subject then decelerates to zero velocity when he thinks that the terminal force has approached the assigned value. An observation of the actual terminal force applied to the rear force lever (point F') shows that the subject exceeded the upper tolerance limit for this cycle. The subject is signaled that he is within the tolerable force range by a buzzer which sounds whenever a force is within the acceptable limits. The subject then begins the return cycle to the front force lever (from point F).

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Designing Control Systems In Work Measurement

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A LARGE part of Industrial Engineering effort is devoted to "Work Measurement." This is put in quotes here since the activity as practiced is more than just "measuring" the time associated with some particular work activity. In practice the engineer also selects appropriate characteristics of the operation to be time-measured, he improves methods and procedures where possible, and he sets up a control system for scheduling and, in the case of incentive standards, pay whereby countable output can be related to time input. In other words, "Work Measurement" really means a complete study and description of the operation.

This analytical process has two goals, both mentioned above:

1. *Improvement* of the operation: using less time, with less fatigue, more safety and generally more satisfaction.
2. *Control* of the operation: knowing the labor content such that future labor requirements can be *predicted* from estimates of future production requirements.

The first goal, higher productivity and greater humanization, is usually sought through work simplification, methods engineering, economic motivation of incentive standards, and other types of motivation. The second goal, control, is sought through a quantitative description of the operation relating production and labor time. This control is desirable since it usually means better utilization of the individual operation with regards to the entire plant—that is, control usually results in higher *overall* improvement.

Mathematics and statistics are fundamental to the effective realization of these goals. In Statistical Quality Control, Experimental Design, and Operations Research, very powerful techniques have been developed that can be used in "Work Measurement." This paper proposes to discuss some of these mathematical and statistical techniques and their role in designing the measurement and analysis of work. Primary attention is paid to the second goal, seeking a control system, although some of these techniques *can* help in selecting better methods and better procedures for improving the individual operation. The area of motivation, such as incentive standards, is not within the scope of this discussion. Mathematical and statistical techniques have not yet been developed—if, indeed, conceived—for characterizing the motivation of human behavior in industrial production operations.

In seeking a control system for an industrial operation, there are four factors of primary importance: 1. The decision tolerance; 2. the decision period; 3. the mathematical model of the operation; 4. the measurement process. These four factors interact with each other and with outside factors in a fashion that can be best described by a small example. Consider a simple repetitive operation of assembling a bolt and nut. Now, the time to do 100 such assemblies would be arrived at by multiplication:

Average time per assembly $\times 100$ = total time for 100 assemblies say, .2 minute $\times 100 = 20$ minutes

This is a trivial calculation, of course, but it can be expressed mathematically with the following substitutions:

$\overline{X}$ = average time = .2 minute
N = production of assemblies = 100
T = total time
$T = N\overline{X} = (.2) \times (100) = 20$ minutes

where $T = N\overline{X}$ is the *mathematical model* for the operation.

Now suppose the time to do an assembly, X, is subject to quite a bit of variation due to, say, discarding defective bolts, as well as to chance operator variation in performing the assembly. If we wished to come to a decision as to the labor content for every 100 assemblies we might run into trouble using $\overline{X}$—the average time per assembly. Our estimates might vary from reality by more than some acceptable tolerance. We would continuously estimate 20 minutes when the actual times to do successive groups of 100 assemblies might vary from, say, 15 to 25 minutes. (This can be treated statistically, of course, but that will come later. The point here is to identify the problem in general terms.) The greater the variation in time per assembly, the greater the variation in the time for 100 assemblies.

If we wanted to make an estimate that was not more than 1 minute off in each direction, we would then have to analyze the operation further to see what was causing the variation. One suggestion as to an assignable cause is the handling of defective bolts. That is, some assemblies require picking up and discarding a defective bolt before making a good assembly. Suppose we decided to *count* such defective bolts in the future and associate a *time* with the handling of defective bolts. This is referred to as "selecting another unit of measure" but, more fundamentally, it is changing the mathematical model. Our calculation now for the time to do 100 assemblies is:

Time for 100 assemblies

= time for 1 good assembly $\times$ 100 assemblies + time for handling a defective bolt $\times$ number of defective bolts for the 100 good assemblies.

If $\overline{X}_1$ = average time for 1 good assembly
N_1 = number of completed assemblies = 100 in this case
$\overline{X}_2$ = average time for handling a defective bolt
N_2 = number of defective bolts for the 100 good assemblies
T = total time

then the mathematical model becomes:

$$T = \overline{X}_1 N_1 + \overline{X}_2 N_2$$

Whether or not successive T's, estimates of time for the 100 assemblies, would more closely match *actual* T's than before would be a measure of this model's success. If it still was inadequate and if the residual variation was still too high, more study would be required to develop a more extensive mathematical model.

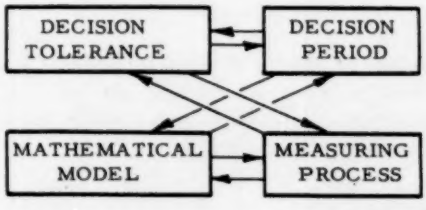
We have just demonstrated one interacting effect—that of the mathematical model on the decision tolerance. Given a decision tolerance and a decision period—in this case within 1 minute over 100 assemblies—the engineer can seek a sufficiently descriptive mathematical model for his control system by being aware of the variability in the system. This is essentially a statistical problem as will be reviewed shortly. Note, however, that it may not have been possible to expand the model. Supposing, in making an assembly, variability existed due only to chance operator causes such as widely differing methods from cycle to cycle. From the point of view of devising a control system, on the *existing* operation, these method differences are not countable and the engineer would have to conclude *that either the decision tolerance and/or the decision period needed modification*. Something would have to give.

Widening the decision tolerance, of course, is an obvious device. If the system can only give you a 2 minute tolerance and you now decide to accept 2 minutes instead of 1 minute, then the problem is resolved. Lengthening the decision period may not be so obvious to those unfamiliar with statistics but it is reasonable when studied. By shifting from 100 to 200 assemblies as the decision period one would reduce the effect of the individual assembly variation. That is, relative to some average time for 200 assemblies, the *actual* times for successive groups of 200 assemblies vary less than that same process with 100 assemblies. Put in even different words—the average times for groups of 200 assemblies bounce around less than the average times for groups of 100 assemblies.

The above demonstrates the interaction between the mathematical model, the decision tolerance, and the decision period. The latter two are not always as inviolate as their name implies although they often exist as policy. In incentive standards situations, for instance, it may be policy (tradition) to post earnings (per cent effectiveness) daily and to require that there be no more than (say a 5% risk of no more than) plus-or-minus 3% variation for reasons beyond operator control (such as the occurrence of defective bolts mentioned above). Lengthening this decision period to one week might not be impossible if pay checks are issued weekly. Similarly in production programming arriving at an appropriate decision period depends on such factors as the mobility of the labor force, the degree of refinement required in meeting deadlines, and the degree of refinement required in cost estimating. It isn't the purpose of this paper to explore the specification of the decision period and the decision tolerance but to point out the relationship of the operation itself and these constraints.

One other possibility in this triangle exists—changing the operation itself. Work simplification, methods engineering, training, and the improvement of product quality into the operation might all be possible ways of reducing methods differences. Of course, one traditional way is to use the time associated with a particularly good method as a "standard" and as a base for premium production pay. This and the other approaches are only successful from a control point of view if they achieve reduced variation and, therefore, better control. Incidentally, in such situations, wide unexplainable variation, *pay* control for incentive standards can be brought within tolerance by a fixed allowance.

So, we have described three of the four factors and some of the ways they interact. The decision tolerance and the decision period may require a mathematical model impossible to obtain with the chance variation in the operation. Changing the decision period and/or the decision tolerance and/or the operation can bring things into a compatible state. The fourth factor, the measurement process, has to do with the engineer's activities. In our bolt and nut example, for instance, having decided to test the mathematical model $T = \overline{X}_1 N_1 + \overline{X}_2 N_2$ we would then be required to obtain, by time measurement, $\overline{X}_1$ and $\overline{X}_2$. Suppose X_2, the time to handle a defective bolt, varied greatly and therefore required an enormous number of observations for adequate precision. This data collection cost might well cause some second thoughts regarding the use of this expanded model and/or suggest reappraisal of the decision period and/or the decision tolerance. So, the measurement process—either because of its sample size requirements or its difficulty in just getting a time measurement—can interact with these other factors. Thus we have completed the circuit and can summarize this qualitative description of the designing of a work control system with a diagram:



To this point, the description has been purely qualitative. Let us now consider some statistical and mathematical techniques that can aid in the analytical process. We'll start with the more familiar area of designing the measuring process and then move on to selecting the

mathematical model. In this part of the discussion, the purpose is to indicate *where* some well-known procedures can be brought to bear in some, not all, phases of Work Measurement. These techniques will be described by name and by characteristic formulas easily recognizable to those familiar with statistics. References containing complete information on why these procedures work are noted and suggested for those unfamiliar with statistics who want to know what goes on "under the hood."

DESIGNING THE MEASURING PROCESS

Measurement is the business of getting a number which reflects what is going on. The measuring process has two problems facing it: 1. Accuracy—its representativeness; 2. Precision—its ability to reproduce measurement values. Accuracy depends on the typicalness of the sample that is selected. Random sampling guided by random number tables is an accepted manner of getting a random sample. This subject is well covered in statistical texts (11) and, with regard to Work Measurement in (1), (3), and (6).

Precision depends on sample size. Here, specific statistical guides are available and are covered in the indicated references.

STOP WATCH WORK MEASUREMENT

SIMPLE REPETITIVE CYCLE

The approximate 95% confidence interval formula in variables measurement is $2s_{\bar{X}}=2s/\sqrt{N}$. This can be simplified by using $\overline{R}/d_2$ in place of s, standardizing on sub groups of 4, ($N=4$, so $d_2=2.059$) and using a per cent precision value

$$E = \frac{2s_{\bar{X}}}{\overline{X}}\,100.$$

The simplified formula:

$$E = \frac{97\overline{R}}{\overline{X}\sqrt{N}}$$

where E = per cent precision
$\overline{R}$ = average range from groups of 4
$\overline{X}$ = average cycle time
N = sample size

The engineer can specify E, use a preliminary study, of, say, 16 observations to get $\overline{R}$ and $\overline{X}$, and solve for N. Or he can, knowing N, $\overline{X}$ and $\overline{R}$ from a study, solve for E. A nomograph for this is displayed in (1).

COMPLEX CYCLE

Not all repetitive operations are so simple, however. Consider one with this mathematical model:

$$\overline{X}_T = \overline{X}_A + \overline{X}_B/\overline{F}_B$$

where $\overline{X}_T$ = average total time per piece
$\overline{X}_A$ = average time per piece
$\overline{X}_B$ = average tray handling time per piece
$\overline{F}_B$ = average pieces per tray

As before, the interest is in the per cent precision

$$E = \frac{2s_{\bar{X}_T}}{\overline{X}_T}\,100.$$

A simplified formula for obtaining E has been developed (1)[1] considering that

[1] The expression

$$\overline{X}_T = \overline{X}_A + \frac{\overline{X}_B}{\overline{F}_B}$$

is of the form

$A = B + C$ wherein B and C are independent.

It follows, then, that

$$s_A^2 = s_B^2 + s_C^2$$

and that

$$s_{\bar{X}}^2 = s_{\bar{X}}^2 + \frac{s_{\bar{X}_B}^2}{F_B}$$

The expression

$$\frac{s_{\bar{X}_B}^2}{\overline{F}_B}$$

is of the form

$$\frac{s_{\bar{C}}^2}{\overline{D}} \text{ the variance of the quotient } \frac{\overline{C}}{\overline{D}}$$

It is generally given that, see, for instance, (5, p. 124)

$$\frac{s_{\bar{C}}^2}{\overline{D}} = \frac{s_{\bar{C}}^2\overline{D}^2 + \overline{C}^2 s_{\bar{D}}^2}{\overline{D}^4}$$

which can easily be developed as follows:

$$\text{let } y = \frac{\overline{C}}{\overline{D}}$$

differentiating

$$\Delta y = \frac{\overline{D}\Delta\overline{C} - \overline{C}\Delta\overline{D}}{\overline{D}^2}$$

$$(\Delta y)^2 = \frac{\overline{D}^2(\Delta\overline{C})^2 - 2\overline{D}\overline{C}\Delta\overline{C}\Delta\overline{D} + \overline{C}^2(\Delta\overline{D})^2}{\overline{D}^4}$$

considering expectations

$$E(\Delta y)^2 = \sigma_y^2$$
$$E[\overline{D}^2(\Delta\overline{C})^2] = \overline{D}^2\sigma_{\bar{C}}^2$$
$$E[-2\overline{D}\overline{C}\Delta\overline{C}\Delta\overline{D}] = \text{cov } \overline{C}\overline{D} = 0 \text{ when } C \text{ and } D \text{ are independent}$$
$$E[\overline{C}^2(\Delta\overline{D})^2] = \overline{C}^2\sigma_{\bar{D}}^2$$

resulting in

$$\sigma_y^2 = \frac{\overline{D}^2\sigma_{\bar{C}}^2 + \overline{C}^2\sigma_{\bar{C}}^2}{\overline{D}^4}$$

This assumes that $\sigma_{\bar{D}}^2$ is relatively small, which is quite reasonable in work measurement situations.
Substituting

$$\frac{s_{\bar{X}_B}^2}{\overline{F}_B} = \sigma_y^2 = \frac{\sigma_{\bar{C}}^2}{\overline{D}}$$

$$\frac{s_{\bar{X}_B}^2}{\overline{F}_B} = \frac{s_{\bar{X}_B}^2\overline{F}_B^2 + s_{\bar{F}_B}^2\overline{X}_B^2}{\overline{F}_B^4}$$

$$s_{\bar{X}_T}^2 = s_{\bar{X}_A}^2 + \frac{s_{\bar{X}_B}^2}{\bar{F}_B}$$

and using the customary expression for the variance of a quotient (5) so that the second term becomes

$$\frac{s_{\bar{X}_A}^2 \bar{F}_B^2 + s_{\bar{F}_B}^2 \bar{X}_A^2}{\bar{F}_B^4} .$$

The simplified formula is

$$E = \frac{97R^*}{\bar{X}_T\sqrt{N_A}}$$

with E and $\bar{X}_T$ as defined above and N_A the observations on the main cycle times X_A. $R^* = \bar{R}_A(1+.15u)$ where $\bar{R}_A$ = average range from groups of 4 X_A's and u = the number of auxiliary elements such as B in this case ($u=1$, of course, in this case). As with the simple cycle, the engineer can specify a desired E, get $\bar{X}_T$ and R^* from a short preliminary study, and solve for N_A. Or, having N_A, $\bar{X}_T$, and R^* from a study, solve for E. The nomograph of (1) also works here.

WORK SAMPLING (ATTRIBUTES MEASUREMENT)

SIMPLE CASE OF % BUSY

The approximate 95% confidence interval formula in attributes measurement is

$$2s_{\bar{P}} = \frac{2\sqrt{\bar{P}(1-\bar{P})}}{\sqrt{N}} .$$

In work sampling, $\bar{P}$ becomes the per cent busy, $2s_{\bar{P}}$ the precision expressed as an absolute per cent plus-or-minus interval, and N the number of busy-not busy observations. The engineer can obtain $\bar{P}$ from a short preliminary study—say 80%, specify $2s_{\bar{P}}$—say 4% or from 76% to 84%, and solve for N which would be 400 in this case. Or, having $\bar{P}$ and N from a study, he can solve for $2s_{\bar{P}}$. The nomograph of (3) handles this calculation.

COMPLEX CASE

Quite often the engineer is interested in breaking down per cent busy into 2 or more classifications. The mathematical model becomes: total per cent busy = $\bar{P}_t = \bar{P}_1 + \bar{P}_2 + \cdots \bar{P}_i \cdots$ etc. with the P_2 values varying. A simplified formula has been developed in (9)[2] solving for:

$$E = \frac{200}{(\bar{P}_1 + \bar{P}_2)\sqrt{N}} \cdot \sqrt{\frac{\bar{P}_1'^2(1-\bar{P}_1)}{\bar{P}_1} + \frac{\bar{P}_2'^2(1-\bar{P}_2)}{\bar{P}_2} - 2P_1'P_2'}$$

In a multi-element study with many $\bar{P}_i$ values, $\bar{P}_1$ = the $\bar{P}_i$ value with the *highest relative upward shift* (maximum $\bar{P}_i'/\bar{P}_i$) and $\bar{P}_2 = \bar{P}_t - \bar{P}_1$. A $\bar{P}_i'$ value is an estimate of the maximum probable $\bar{P}$ value. So, given current $\bar{P}_i$ values from a preliminary study, estimating $\bar{P}_i'$ values from the study or history, and specifying E—*the desired precision on some future possible mix of proportions*; the engineer can solve for N—the sample size to take *now*. Or, given N, $\bar{P}_i$ values, and $\bar{P}_i'$ values, he can solve for E.

---

[2] Consider the simple case wherein $\bar{P}_t = \bar{P}_1 + \bar{P}_2$

Let T = total busy time of the study

h = total labor time associated with the study (i.e., 10 men for 1 week would be: $h = 10 \times 40 = 400$)

$T = A_1u_1 + A_2u_2$

where $u_1 = N\bar{P}_1$ N = total number of observations

$u_2 = N\bar{P}_2$ $\bar{P}_1, \bar{P}_2$ = Decimal per cent busy on operation 1, 2

$A_1 = A_2 = \frac{h}{N}$

per Cramér (6, p. 318)

$$\sigma_T^2 = A_1^2\sigma_{u_1}^2 + A_2^2\sigma_{u_2}^2 + 2A_1A_2(\sigma_{u_1u_2})$$

where: $\sigma_u^2 = N\bar{P}_1(1-\bar{P}_1)$

$\sigma_{u_2}^2 = N\bar{P}_2(1-\bar{P}_2)$

covariance $u_1u_2 = N\bar{P}_1\bar{P}_2$

Considering a *future* proportion where $\bar{P}_1' = .6$ $\bar{P}_2' = .2$ $\bar{P}_t' = .8$

Let $T = T' = A_1'u_1 + A_2'u_2$

$$A_1' = \frac{\bar{P}_1'}{\bar{P}_1}\frac{h}{N} \qquad A_2' = \frac{\bar{P}_2'}{\bar{P}_2}\frac{h}{N}$$

$$s_T^2 = A_1'^2s_{u_1}^2 + A_2'S_{u_2}^2 + 2A_1'A_2'(\sigma_{u_1u_2})$$

$$s_T^2 = \left(\frac{\bar{P}_1'}{\bar{P}_1}\frac{h}{N}\right)^2 N\bar{P}_1(1-\bar{P}_1) + \left(\frac{\bar{P}_2'}{\bar{P}_2}\frac{h}{N}\right)^2 N\bar{P}_2(1-\bar{P}_2) - 2\frac{\bar{P}_1'}{\bar{P}_1}\frac{\bar{P}_2'}{\bar{P}_2}\left(\frac{h}{N}\right)^2 N\bar{P}_1\bar{P}_2$$

$$s_T^2 = \frac{h^2}{N}\left[\frac{\bar{P}_1'(1-\bar{P}_1)}{\bar{P}_1} + \frac{\bar{P}_2'(1-\bar{P}_2)}{\bar{P}_2} - 2\bar{P}_1'\bar{P}_2'\right]$$

95% confidence interval as a relative %

$$\frac{2s_T}{T}100 = E = \frac{200h}{T\sqrt{N}}\sqrt{\bar{P}_1'^2 \text{ etc.}}$$

but $T = (\bar{P}_1 + \bar{P}_2)h$

$$\therefore E = \frac{200}{(\bar{P}_1 + \bar{P}_2)\sqrt{N}}\sqrt{\frac{\bar{P}_1'^2(1-\bar{P}_1)}{\bar{P}_1} + \frac{\bar{P}_2'^2(1-\bar{P}_2)}{\bar{P}_2} - 2\bar{P}_1'\bar{P}_2'}$$

Thus, having a sampling estimate of $\bar{P}_1$ and $\bar{P}_2$

— an estimate of $\bar{P}_1'$ and $\bar{P}_2'$

— specifying E (the precision on some *future* mix)

— solve for N—the number of observations to take *now*

Or

having N—solve for E

Note—with several proportions

Pa Call the proportion with highest *relative upward shift*

Pb $= \bar{P}_1$ and $\bar{P}_2 = P_t - \bar{P}_1$

Pc

Pd That is, $\max \frac{\bar{P}_i'}{\bar{P}_i} = \bar{P}_1$

⋮

SELECTING THE MATHEMATICAL MODEL

INTERNAL VARIATION

Recalling the earlier example with the model $T = N\overline{X}$, the engineer must be satisfied that this "one unit of measure" model is all right. A first check can treat the average decision period production as a sample and see if the 95% confidence interval on that sample is within some specified tolerance. He essentially solves for a value we'll call

$$E_r = \frac{2s_{\overline{X}}}{\overline{X}}\,100 \quad \text{where} \quad 2s_{\overline{X}} = \frac{2s_{\overline{X}}}{\sqrt{N_P}}$$

and N = decision period production. If $E_r > 5\%$, say, then one unit of measure is insufficient. The expanded model $T = \overline{X}_1N_1 + \overline{X}_2N_2$ could then be tested and so forth. The simplified formula for this using groups of 4, $\overline{R}$, and d_2 is

$$E = \frac{97R^*}{\overline{X}_T\sqrt{N_P}}$$

and it can be handled on the nomograph of (1).

BETWEEN DECISION PERIOD VARIATION

Besides short term sampling variation in the operation, there may be inherent decision-period-to-decision-period variation. Determining whether or not this variation (call it $2s_d$) is greater than some specified tolerance (say 5%) can be a tricky problem. One way is to observe several decision periods completely and get a sample between-decision-period standard deviation. This can be expensive, however, and using the notion of variance components would appear much better. In observing samples from several decision periods (at least 2) the following relationships would prevail:

Total variance = sampling variance + decision period variance

$$s_T^2 = s_s^2 + s_d^2 \quad \text{and} \quad s_d^2 = s_T^2 - s_s^2.$$

Total variance, s_T^2, is observed; sampling variance, s_s^2, can be calculated by pooling decision period variances; and one can compute s_d^2—the variance associated with the decision period variation that would prevail if there was no sampling. Components of variance are covered quite extensively in (11).

The interest in this case is in

$$E_v = \frac{2s_d}{\overline{X}_T}\,100.$$

Getting s_d^2 and then s_d from a preliminary study along with $\overline{X}_T$ the engineer can match the computed E_v against the tolerance. If it's greater, then more units of measure are needed and an expanded model can be tested. There is a possibility, of course, that sample size is insufficient for any decision, $s_s^2 > s_T^2$, and further study with a larger sample is then required.

MULTIPLE REGRESSION—VARIABLES

We have talked to date about examining progressively expanded models until the residual variation is within tolerance. The tacit assumption has been that times for each newly considered auxiliary element of the operation were obtainable by direct observations. This is not always possible such as in the case of packaging operations. The time to package one type of item among many types being packaged is often thoroughly mixed in with and dependent on the times of packaging these other types. All that is known about the operation is that for each package some total time has been put into packaging and a variety of countable types of items packaged. The question is: How many of the item types should be counted for an adequate control system? The question, of course, only comes after having decided by previous tests that just counting packages is insufficient.

This is a problem in multiple linear regression. Consider this typical example wherein from a short preliminary study times for packaging one order (X_T) have been obtained along with the associated counts of the three types of items being packaged (N_1, N_2, N_3).

Time to Package One Order X_T	Number of Items of Type 1 N_1	Number of Items of Type 2 N_2	Number of Items of Type 3 N_3
.92	11	4	5
.67	1	13	1
.47	4	5	0
.	.	.	.
.	.	.	.
.	.	.	.

The results of a multiple regression analysis (see (10) and Footnote 1) on the complete data from studying 16 orders showed the following:

$$X_T = \overline{X}_0 + \overline{X}_1N_1 + \overline{X}_2N_2 + \overline{X}_3N_3$$

i	$\overline{X}_i$	Decision Period (Daily) Variation (E_r)
	.541	6.0%
0	.318	—
1	.852	5.9
2	.813	1.2
3	.013	.9

$$X_T = .318 + .852N_1 + .813N_2 + .013N_3$$

Note the recording of the residual variation (S_{y_i}) opposite the associated regression coefficient ($\overline{X}_i$). This is phrased in terms of chance variability of a decision period (daily) sample N_P:

$$E_r = \frac{2S_{y_i}}{\overline{X}_T\sqrt{N_P}}$$

If $\pm 5\%$ were satisfactory for E_r then item type 3 could be dropped. Subjecting the final study to a two type regression analysis would probably generate a model satis-

factory for the control system. Incidentally, sample size can be approximated in such cases by using the previous confidence interval formula

$$E = \frac{2S_{y_i}}{X_T\sqrt{N}}$$

where S_{y_i} is the cut-off S_{y_i} value (S_{y_3} in this case). Also note that E, between decision period variation, can be examined by analyzing the individual regression residual values.

MULTIPLE REGRESSION—WORK SAMPLING

The same approach can be used with work sampling results from a non-repetitive operation study. A work sampling per cent busy can be easily converted to a time estimate. Consider these data from a study of the electrical trades:

Job No.	Per Cent Busy		Total Job Time (from job tickets)		Direct Labor Time
1	73	×	52 hours	=	38 hours
2	65	×	171 hours	=	111 hours
.	.		.		.
.	.		.		.
.	.		.		.

The time inputs for each job studied and the associated counts for each job of the conduit sections, footage of small size conduit installed, and footage of large size conduit installed were the ingredients for the multiple regression analysis:

Job No.	Work Sampling Hours X_T	Conduit Sections N_1	Small Size Conduit Feet N_2	Large Size Conduit Feet N_3
1	7.3	17	13	72
2	76.7	180	804	241
3	9.0	29	0	62
4	26.7	73	399	0
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

$$X_T = .25 + .12N_1 + .04N_2 + .08N_3$$
$$2S_{y_3} = .3 \text{ hours or } \pm 4\% \text{ on the average job}$$

Fewer or additional units of measure could have been considered but this residual variation was felt to be correct.

QUEUEING MODELS

Service type labor is often analyzed in Work Measurement. This is labor that is on a demand basis—labor of an indirect or non-repetitive type whose services are constantly on *call* but only intermittently, randomly used. Machine-breakdown repairmen are in this category and are increasingly important as automation moves into the picture. Oddly enough, this control problem has become more prominent as the use of work sampling has increased. For work sampling has effectively disclosed, time and again, the fact that a service crew is inactive part of the time. But this information is not enough. Arbitrary crew reduction merely creates machine delays on those occasions when several machines demand service at once, possibly as costly as idle labor.

The idle labor can be considered as labor "queueing up" or waiting in line to service machines. Machine delays can also be considered as machines "queueing up" to be serviced. This queueing problem can be resolved by the use of queueing theory, a segment of probability theory, which permits the engineer to predict the effect of various crew sizes on the idle labor and machine wait time characteristics.

Here is a simple example. At our Kodak Park Works, the Kodacolor Printing Room, at a particular production level, had a crew of 3 handlers per shift supplying and inspecting magazines of photographic paper for 28 printers. Since these magazines had paper of varying length and the printers had varying speeds, the demand pattern for supplying magazines was quite unsystematic. Printer magazine servicing occurred randomly and was unpredictable. The inspection of magazines was routine, however. They could be inspected at leisure and stored until needed. So the problem was one of planning the work load, a portion of which was randomly demanded.

Work sampling results showed the three men were busy 47% of the time (2) and (12).

The obvious move to reduce the crew to 2 men and experience about 71% activity (47%×3/2) was not automatically the right move owing to the demand nature of their work. Queueing theory had to be employed to evaluate the effect of such a possible move.

The engineer procured the following basic information about the operation:

1. $m = 28$ = number of printers under consideration
2. $r = 2$ = number of servicemen under consideration
3. $a = \lambda/\mu = .0246$

Where $\lambda = 3.1$ minutes = the average time for a handler to supply a magazine to a printer.

$\mu = 125.8$ minutes the average run time of a printer

The average magazine change time (λ) was obtained from regular stop watch studies. The average printer run time (μ) was obtained from printer logs where the paper footage for each run was recorded. Knowing printer speed, time for each run and the average run time were simple to compute. With this basic information, the engineer used a table based on this mathematical model (1) and (12):

$$\frac{P_n}{P_0} = \frac{m!a^n}{n!(m-n)!} \quad r \geq n \geq 0 \qquad P_n = P_0\left(\frac{P_n}{P_0}\right)$$

$$= \frac{m!a^n}{r!r^{n-r}(m-n)!} \quad r \leq n \qquad i = \frac{\sum_{n=r+1}^{m}(n-r)P_n}{m} \quad r \leq n$$

$$P_0 = \frac{1}{\sum_{n=0}^{m}\frac{P_n}{P_0}} \qquad = \frac{\sum_{n=0}^{r}(r-n)P_n}{r} \quad n \leq r$$

where P_n = the probability of n machines being down together (requiring service)

$$(P_0 = P_n \text{ for } n = 0)$$

i = the average machine wait time expressed as a decimal fraction

f = the average serviceman wait time expressed as a decimal fraction

He determined the values as follows:

$i = .0033$ or about $.3\%$
the average fraction wait time per printer

$f = .6353$ or about 63.5%
the average fraction wait time per handler

Thus he could say that if the crew were reduced to 2 men, the delay introduced by such a reduction (delay due to interference) would be about $.3\%$. The handler idle time, of course, would be much less than 63.5% in actuality owing to the inspection portion of the job; the work sampling 2-men crew estimate of 71% busy, or 29% idle, would still hold. Queueing theory only considered the random demand portion of the operation. A final check on the applicability of the model had to be made. The computed i and f values for the *observed* (3 man crew) situation were compared to the *actual sampling* results on i and f:

	Observed	*Calculated by the Model*
i	0	$.03\%$
f	$73\% \pm 5\%$	75.6%

These observed data fitted the model for all practical purposes meaning the 2-man crew prediction was quite likely valid.

There are other machine interference models (4) (13) handling other types of situations. Note that here we can achieve improvement, perhaps, in the operation as well as gain control. Production level shifts (more machines) can be considered in this model and, for a desired i and f balance, the optimum crew size assigned. The "unit of measure" here is essentially the number of machines and crew size is the dependent variable. The function is not a simple linear one but can be tabulated for scheduling use. Many "indirect" type operations are susceptible to this type of control where the unit of control is an individual —not some portion of his or her time as in simple repetitive cycle.

LINEAR PROGRAMMING AND HANDLING

Considerable attention has been paid to the "indirect" operation of handling. Work sampling has made it possible to measure this activity but activity measures do not always resolve the problems of either control or improvement. The linear programming model is an effective aid in determining what handlers are to be where and at what time to meet specified service requirements. Consider this handling problem with the following demand matrix of loads per hour:

		From		
	Area →	1	2	3
	↓			
	1	0	12	4
To	2	6	0	2
	3	5	1	0

These areas are equidistant. If a handler can move 6 loads per hour then the ideal number of handlers in all is

$$\frac{6 + 5 + 1 + 12 + 4 + 2}{6} = \frac{30}{6} = 5.$$

To set up a good method of handling (not necessarily the "one best method" but a good one) by linear programming to achieve one hour service some possible routes can be listed—say 10 to start with. And associated with each route is an unknown number of handlers:

Route		*Number of Handlers*
I	1, 2, 1, 2, 1, 2, 1	a
II	1, 3, 2, 1, 2, 3, 1	b
III	1, 3, 1, 3, 1, 3, 1	c
	etc.	

Now, for each element in the demand matrix an inequality can be written. For instance, element 1, 2 requires 6 loads per hour be moved. Route I can move 3 loads for each handler on that route and Route II can move one load for each handler it has. So, $3a+b \geq 6$ if these 3 routes are the only ones considered. Other routes along the initial 10 might have a 1, 2 segment so the general inequalities for this and all elements become:

Element	
1, 2	$3a + b + \cdots\cdots \geq 6$
1, 3	$b + 3c + \cdots \geq 5$
3, 1	$b + 3c + \cdots \geq 4$
	⋮

and we would want to minimize the total number of handlers $Z = a+b+c+ \cdots$. These inequalities along with the function to be minimized constitute a linear programming problem. Solution techniques, some underlying ideas in linear programming and a report on a somewhat similar application with railway freight cars are contained in references (7) and (8).

A few comments on this type of application—the solution to the above may not be the best since all possible routes have not been considered. However, it's very easy to write down any number of additional routes and assess their improvement, if any. Also, high variation in demand may upset service. Each channel with a given schedule can be examined with queueing theory and the channel especially loaded if the waiting line is too long. Note that

here, again, both improvement and control are achieved. As demand increases or changes, the queueing model suggests the appropriate rearrangement of handling assignments to meet this new demand. The relationships can be pre-computed and tabulated. The simple linear relationship of production×average time no longer holds.

SUMMARY

There are four basic factors affecting the design of a work control system:

1. *The decision tolerance:* how close you want to be in predicting or stating labor content.
2. *The decision period:* how frequently you want to make a prediction or statement.
3. *The mathematical model:* symbolic portrayal of the elements in the operation.
4. *The measuring process:* what the engineer has to do to get the numbers for the mathematical model.

These factors all interact and the Industrial Engineer seeks a balance among them every time he undertakes a Work Measurement study. This balancing is usually intuitive and expensive in that poor decisions require later analysis to repair the situation. Statistical and mathematical techniques do exist to help the Industrial Engineer handle these problems with considerably greater effectiveness. As demonstrated in Quality Control and in Operations Research, the payoffs can be huge on both cost cutting for the plant's operations and reduced engineering time. A vast gap exists, however, between the knowledge available and the knowledge used. In proportion as we all team up and tap the immense theoretical power that is just out of reach now, we can move toward our mutual goal of bringing more science to management

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Ready for Anything

TO BE "ready for anything" is a large order. The man who specializes concentrates, of necessity, on a relatively narrow spectrum of knowledge. To be an "expert" implies an understanding of specific subject matter which is deeper than that of the general practitioner.

The Electrical Engineer these days is generally an expert on the behavior and control of electrical energy; he may know little about specifying materials or managing men. A Metallurgical Engineer may know much about one segment of materials—metals—and at the same time have only a sketchy knowledge of energy, and managing people.

A Mechanical Engineer undoubtedly knows something about both energy and materials, but may know little about man himself.

The Industrial Engineer must understand the modus operandi of both the physical and the behavioral sciences. He must know that it is one thing to design and predict the performance of a machine, but quite another to predict the reactions of people toward his new design.

The machine performance is largely based upon the physical sciences, but the forces which determine the reactions of people are generally in the area of the behavioral sciences.

The Industrial Engineer must understand economics and cost accounting. A casual glance tells the observer that the Industrial Engineer should have a composite education, including some facets of the education found in other branches of engineering, plus psychology, plus sociology, plus economics, plus cost acounting. While a capable man with such a background would be a rare jewel, he also would be much too old when he completed his schooling to be just starting a career.

So the education of the modern Industrial Engineer is a compromise. It must be broad, but it also must impart depth of understanding in certain limited fundamental areas, to the exclusion of strictly vocational courses.

More importantly, in order to be "ready for anything," the modern Industrial Engineer must be quick in perception, wise in reasoning, and responsive to the pursuit of new knowledge. He must have intelligent and directed adaptability whenever a need for change is clearly evident. He must continually be a scholar if, in his lifetime, he is to master the full scope of his profession.

The earning curve of a truly professional engineer continues to rise until he reaches retirement age. The earning curve of a tradesman tends to level off within a decade after he has "learned his trade." So the tangible rewards of true professionalism are real, and the intrinsic rewards of the satisfactions which result from a feeling of competence—being ready for anything—are even greater.

The modern professional Industrial Engineer is broadly based in both the physical and behavioral sciences. By nature he has an insatiable curiosity; he is an indefatigable scholar. By these traits, and as he matures, he extends the breadth of his knowledge and the depth of his understanding of specific phases of his work.

Today's professional Industrial Engineer will find the horizons of his opportunity much more extensive than even the wildest dreams would have indicated when AIIE was founded a little less than a decade ago.

institute interests

CALENDAR

January 6, 7 and 8, 1958: Fourth National Symposium on Reliability and Quality Control; Hotel Statler, Washington, D.C. Write Richard M. Jacobs, RCA Bldg. 108-2, Moorestown, New Jersey.

January 27-February 6, 1958: Engineering and Management Course: University of California, Los Angeles. Write Administrative Staff, Engineering and Management Program, U.C.L.A.

February 15, 1958: Cost Control Clinic (Phil Carroll); Skirvin Hotel, Oklahoma City, Oklahoma. Write D. K. Andrews, Jr., 2420 N.W. 54th, Oklahoma City 12, Oklahoma.

March 17-21, 1958: Fourth Nuclear Engineering and Science Conference; Chicago, Illinois. Write Atomic Exposition, 117 South 17th Street, Philadelphia 3, Pennsylvania.

CLEVELAND CHAPTER

The inaugural meeting of the 1957-58 AIIE program year was held on October 8 at Hotel Cleveland. Dr. Irwin P. Lazarus discussed "Developing Standard Time Data." Dr. Lazarus is Asst. Director on the Control Management Staff of the American Colortype Company of Chicago. He is also Vice President of the Central Region, AIIE.

The speaker has been one of the pioneers in scientific work measurement. He was educated from B.S. to Ph.D. at Purdue University where he served on the faculty for 5 years. The speaker was dynamic in his presentation and excellent in his humor. All in attendance benefited from his explanation of his work measurement system.

Present membership of the Cleveland Chapter is 240. It is the third largest chapter in the Country.

The November meeting was in the form of a "Bosses Night," held on Thursday, November 7, in the Empire Room of the Cleveland Hotel. Mr. Bertrand J. Belda spoke on "Controlling Costs with Budgetary Control." Mr. Belda is a Partner in the firm of Ernst and Ernst and is Director of Management Services for the 32 offices of the Firm's Central District. He has had an extensive background of study and experience in budgeting for cost control, and discussed some of the techniques that can be applied in establishing a flexible budget program, including the work measurements of direct and indirect labor that are essential to sound budgetary control.

DAYTON CHAPTER

For the engineer who aspires to, or is already a member of, management, the art of making decisions assumes great proportions. Recognition of this fact was recently gained when numerous Dayton industries sponsored their most promising supervisors in a management development program on decision making. Our October speaker, Dr. Charles Scheidler, was decidedly one of the highlights of this program. This meeting was held on October 17 in the Mechanical Engineering Building on the University of Dayton campus.

Dr. Scheidler received his Bachelor's and Doctor's degrees in psychology from Washington University, in St. Louis, where he also taught and served as Test Administrator. Before joining the University of Dayton faculty in 1953, he was associated with the American Institute for Research in Pittsburgh. Presently he is a full professor of psychology and a counselor in the U. D. Guidance Center.

DETROIT CHAPTER

Members of the Detroit Chapter have been spending a busy summer planning their first annual chapter conference to be held the latter part of January, 1958. Preliminary plans call for a day and a half meeting with several nationally known personalities as principal speakers. The general theme of the conference is going to be Work Measurement. Although this is the first venture for the Detroit Chapter in holding a conference, present plans indicate that this will be a big affair.

The Program Committee has been busy planning a suitable program of speakers and panels for the coming meeting year. The guest speaker at the September meeting was Central Region Vice President, Professor Harold T. Amrine. This meeting also served as a general session on the Chapter Conference.

In October, Chester Brisley, one of our internationally known members, was principal speaker. He spoke on "Work Sampling Applied to the Executive."

The Detroit Chapter meets each second Tuesday of the month at the Engineering Society of Detroit, Rackham Memorial Building, 100 Farnsworth Avenue. If any members should be in Detroit on our meeting night, they are cordially invited to attend our meetings.

HOUSTON CHAPTER

The first program of the 1957-1958 season was held on September 18 on the subject "Labor's Views Toward Industrial Engineering." The meeting began with an introduction of new officers and of Dr. J. T. Elrod, new Southwest Region Vice President and member of the Houston Chapter.

Speaker for the evening was Mr. Bertram Gottlieb, Staff Industrial Engineer in the Research Department of AFL-CIO National Headquarters in Washington, D.C. Mr. Gottlieb described labor's views concerning work measurement, wage incentives, job evaluation, and other Industrial Engineering techniques which directly affect the worker. Having these subjects analyzed by an Industrial Engineer with labor's viewpoint was exceptionally stimulating and informative.

On November 15, 1957, the Houston Chapter and the Industrial Engineering Department, University of Houston, will sponsor the Third Annual Work Measurement Clinic at the University of Houston. The program for the one-day session, which is open to all interested persons, will be as follows:

SPEAKERS

H. B. Brandt, Associate Director of Industrial Engineering, Procter & Gamble Company, Ivorydale, Ohio, "Multiple

Factor Incentives."

Dr. G. Nadler, Chairman, Industrial Engineering Department, Washington University, St. Louis, Missouri, "Work Measurement Today and Tomorrow."

K. A. Lifson, Consultant Engineer, Dallas, Texas, "Quick and Dirty Techniques for Balancing Crews in Non-Repetitive Work."

Noah Hull, Assistant to Vice President of Manufacturing, Hughes Tool Company, Houston, Texas, "Management Looks at Work Measurement."

PANEL

A panel will discuss questions from the floor, affording participants the opportunity of having their problems analyzed by the "experts."

MODERATOR

S. A. Scharff, District Supervisor of Industrial Engineering, Ernst and Ernst, Houston, Texas.

MEMBERS

Clinic Speakers and

Fred Macy, Chief Industrial Engineer, Shell Refinery, Houston, Texas.

A. B. Keckley, Vice President and General Manager, Texas Electric Steel Castings Co., Houston, Texas.

LOS ANGELES CHAPTER

The September meeting featured a talk by Roger C. Sisson, partner in Canning, Sisson & Associates. Mr. Sisson discussed "How Electronic Data Processing Will Develop Better Industrial Engineering Standards." His talk was well received by the 67 members present. A highlight of the evening was the presentation of Past-President pins to Elwood Buffa, Howard Dickinson, Arthur Chilman, and Arnold Cowan.

The October meeting consisted of a dinner followed by two distinguished speakers. Charles E. Chapel, a member of the State Assembly, addressed the group on: 1. Recent enacted laws pertaining to Engineers, 2. Methods of influencing legislation and 3. Suggestions for obtaining registration for engineers. The second speaker, Mr. A. E. Bogardy, has been Chief Advisor to the French Government on matters of productivity. His topic was "Industrial Engineering, an Important Element of Productivity in Europe."

The November meeting featured Mr. George Sullivan of Benjamin Bordardt & Associates, speaking on "Industrial Engineering and Management Control." A technical clinic and film rounded out the evening.

We meet at the Rodger Young Auditorium the second Tuesday of each month. Industrial Engineers who have moved to this area are invited to attend.

LOUISVILLE CHAPTER

Our new season started with a well attended dinner meeting held on September 18 at Bellarmine College. The principal speaker of the evening was Dr. Ben S. Graham, Director of Methods Research, Standard Register Company, Dayton, Ohio. His subject was "Management Planning—The Industrial Engineers' Challenge." In his presentation, Dr. Graham discussed the growing complexity of modern business, especially with all of the new techniques that are continually cropping up (Operations Research, Linear Programming, Industrial Relations Programs, etc.) and the growing tendency for "empire building" among those factions that specialize in these various techniques. His definition of Management Planning is: the coordination and integration of the functions of management into a smoothly operating management pattern.

Our planned MTM Conference was held on October 4 at Bellarmine College. This conference was a huge success, thanks to the very capable efforts of Ed Thoben and the others who worked with him. After a warm welcome by the Very Reverend Monsignor Raymond J. Treece of Bellarmine, we heard Sterling Durst who gave us a fill-in on the background of MTM.

Mr. Jim Kirchner of the Serge Birn Co. then went into actual detail of explaining and applying MTM data, giving numerous illustrations of movements, problems, limitations, etc.

At luncheon, Mr. Richard Stoll, Executive Secretary of the MTM Association for Standards and Research gave a talk on the application of MTM and its import.

The afternoon contained four laboratory sessions where the participants had a chance to observe the technique of analyzing actual operations and applying MTM Standards.

Our evening speaker was Mr. Tyrus Davis who discussed the aspects and philosophy of employee-employer relations within industry.

The facilities at Bellarmine proved to be ideal for this meeting and we are indebted to the College for the wonderful manner in which they made them available for our use.

MEMPHIS CHAPTER

We had a very unusual meeting planned for the month of October. On Wednesday, October 16, we met at the plant of the Memphis Publishing Co. Our host was Mr. George Sisler of the *Commercial Appeal*. It was most interesting to us as Industrial Engineers to observe the process and methods of publishing a daily newspaper for a city the size of Memphis.

We were given a grand tour of the plant, plus publicity pictures of the group and finally an invitation to the "Press Club" where refreshments and drinks were served at individual expense.

We take this opportunity to congratulate our newly elected officers for the coming year. The following were elected at our September meeting: President—Joe Maloy, Jr.; Vice President—Hancel Bonds; Secretary—Leroy Leslie; Treasurer—Jim Barr.

MIAMI CHAPTER

The Miami Chapter, in conjunction with the Time and Motion Study Institute of the University of Miami, held the Sixth Annual Time and Motion Study and Management Conference on October 31 and November 1 of this year. This meeting, held at the Columbus Hotel in Miami, was directed by Dr. J. P. Lesperance and consisted of seven major speakers in the field of Industrial Engineering and Management.

This conference provided management in the Florida area with an unusual opportunity to develop the talents of its key personnel who are in line for greater management responsibility.

NASHVILLE CHAPTER

The Nashville Chapter of the American Institute of Industrial Engineers received its Charter at the first regular dinner meeting this year on September 9, 1957. The meeting was held at the Highland Crest Restaurant, Nashville, Tennessee. Vice President, Floyd J. Titler, Assistant Chief Industrial Engineer of Republic Steel, presented the Charter to the temporary chapter President, Mr. Warren Gimple.

Permanent officers were elected and installed by Mr. Titler at the September meeting.

The October meeting of the Nashville Chapter was held jointly with the Nashville Chapter of the Society For Advancement of Management. Mr. John Heritage of E. I. du Pont Company, W[illegible]ington, Delaware, the guest speaker, spoke on "Measuring Maintenance Labor."

NASHVILLE CHAPTER OFFICERS. Left to Right: Lawson Lee, Ford Motor Co., Secretary; Warren Gimple, E. I. du Pont, President; Harry Douglas, Temco, Inc., Vice President; William G. Butler, Temco, Inc., Treasurer.

At the November meeting we were very fortunate in having Professor C. G. Johnson of Georgia Institute of Technology as our guest speaker. His topic was "Operations Research" and it was well received by both members and guests.

The Nashville Chapter initiated its first Industrial Engineering Clinic, sponsored jointly by S.A.M., in an all day session at the Highland Crest, Nashville, Tennessee. The Clinic was conducted by Professor C. G. Johnson and provided an opportunity for members and guests representing four area industries to review and study "Work Sampling Techniques."

OKLAHOMA CITY CHAPTER

Our 1957-58 season began with a dinner meeting on September 12 at Patrick's. Ralph M. Ball, architect partner of the firm of Hudgins, Thompson, Ball and Associates, discussed Oklahoma City's Fifteen Year Plan for industrial development. Mr. Ball pointed out the interrelationship and need of expansion and modernization of Oklahoma City's Will Rogers Field in connection with over-all development of desirable industrial sites. Using a specially prepared map of the city's area, he explained the planning behind proposed and in-construction expansions of utilities, expressways, and highways.

The lively question and answer period following Mr. Ball's talk indicated that his subject and presentation was received with considerable interest.

Many of the members of this Chapter are graduates of Oklahoma A. & M. College (now Oklahoma State University), and the fact that Mr. Ball is also serving this year as President of that college's Alumni Association, produced a post-meeting session on football planning and tactics.

PHILADELPHIA CHAPTER

For the first meeting of the 1957-58 season held Wednesday, September 18, the Philadelphia Chapter presented a lecture dealing with the requirements for and value of professional licensing for engineers, particularly as it relates to Industrial Engineering. Mr. Robert E. Derby of Albert C. Wood Associates, Consulting Engineers, was the speaker of the evening. Mr. Derby is Chairman of the Inter-Professional Relations Committee of the Pennsylvania Society of Professional Engineers. He pointed out the fact that a Professional Engineer's license is pre-requisite to practice in some fields of endeavor and that some firms are requiring professional licensing as pre-requisite to employment. This latter practice may spread as the need and value of employing such individuals is recognized. Mr. Derby also stressed the fact that it should be the desire of all engineers to gain professional recognition for their chosen fields and that organizations such as the AIIE should encourage and assist in attaining these aims.

The lecture was followed by a discussion of the requirements in the State of Pennsylvania for a Professional Engineer's license as they relate specifically to the Industrial Engineer. These requirements are specified in the Pennsylvania Professional Engineers Registration Law, Act 367, approved May 23, 1945. A distinction was made between engineering graduates of ECPD (Engineers Council for Professional Development) accredited schools and those without such accreditation. Still another distinction was made between those who have college degrees other than engineering but are currently performing Industrial Engineering functions and those who have no college degree at all. Those graduates of ECPD schools could receive a Certificate of Engineer-in-Training without the necessity of taking an examination while the others have to do so after accumulating equivalent years of experience. The Professional Engineer's license is obtained after passing an examination also. Since the audience consisted of individuals with varying qualifications, the suggestion was made that those interested should address their inquiries to: Miss Rebecca S. Nickles, Secretary, State Registration Board for Professional Engineers, Harrisburg, Pennsylvania.

A brief business meeting was held after this discussion for the purpose of electing a new Chapter Vice President. Mr. Thomas A. Nupp who was elected to this office last spring has left the Philadelphia area. The vacancy was filled by the election of Mr. Luther A. Darmstaetter, Chief Industrial Engineer, Congoleum-Nairn, Inc.

The subject of the season's second meeting, held Wednesday, October 18, was "Warehouse Proportions for Optimum Handling." Mr. Frank M. Ware, Jr., Industrial Engineer, of the Rohm and Haas Company, was the speaker of the evening. He reviewed the factors relating to the design of warehouse buildings and facilities. Mr. Ware related these factors to their application in the design of a recently completed single story warehouse of 25,000 square feet constructed in Chicago.

Some of the specific factors discussed included: the inventory period, type and number of products to be handled, methods of packaging and handling, volume to be handled, present and anticipated, internal transportation facilities, special storage requirements of insurance companies, expected cost, savings and write-off period, and determination of aisle space. The physical size of the building was determined by a mathematical formula which indicated that in this particular application the ratio of length equal to twice the width would result in the most efficient methods of operation. The value of pre-planning was evident from the amount of preparation and research which was devoted to this project.

ST. LOUIS CHAPTER

Linear programming and a foreman training technique have been presented to the St. Louis Chapter in its September and October meetings. Both programs featured audience-involvement and each was received enthusiastically by the membership as the program for the year began.

Mr. Kenneth S. Barten, Associate Professor of Industrial Engineering at Washington University, provided an audience participation session in one technique of linear programming at the September meeting. The attendees worked out, with Professor Barten's direction, a simple machine loading matrix to achieve a profit maximization. A question and answer session concluded the evening's program.

The program for the October meeting was presented by Mr. Lawrence Pallas, a social work director for the Jewish Community Centers Association. Mr. Pallas examined the foreman training

tool, psychodrama, for the benefit of the members present. The technique, in its practical application, involves role playing in which the supervisor or the Industrial Engineer assumes roles of the persons with whom he deals to evaluate their point of view. Mr. Pallas divided those present into four-man groups. Each group probed a representative problem for a specified period of time. The resultant solutions were evaluated for the entire group.

A Midwest Regional Conference-Workshop is scheduled for March 12, 13, and 14 at the Chase Hotel in St. Louis. The conference, sponsored by the St. Louis Chapter, is under the planning of Charles Wicks. Notices of the conference, "Cost Reduction Through Industrial Engineering," will be mailed to all members of midwest region chapters. Nationally recognized authorities in the areas of cost reduction and Industrial Engineering have been secured for the first two days of speaking sessions. The final day of the conference will be an all day workshop.

SAN FRANCISCO-OAKLAND CHAPTER

The San Francisco-Oakland Chapter of AIIE has initiated a T.V. series on Work Simplification under the chairmanship of Mr. Chuck Ebeling. The series—titled "There's Always A Better Way"—is designed to illustrate the fundamental approach employed by Industrial Engineers to increase the productivity and decrease the effort of a job. The series consists of fifteen one-half hour programs over KQED (Channel 9) from 8:00 to 8:30 p.m., P.S.T.

Some of the titles and participants on these programs were: "An Introduction to Work Simplification," Frank Hoffman, C&H Sugar Co., and R. L. Knighton, Columbia-Geneva Steel Div.; "An Introduction to Motion Study," C. W. Ebeling, General Foods Corp., and W. W. Wiesenfeld, Coast Mfg. & Supply; "The Systematic Approach," D. A. Davis, and D. C. Sleeper, both of Columbia-Geneva Steel Division; "Junior Achievement and Work Simplification," J. R. Frost, California-Hawaiian Sugar Refining Co., and F. E. Taylor, Oliver Tire & Rubber Co.

Space limitations prevent listing all the programs in the *Journal*, but such activities deserve considerable recognition for their service to the profession and the community.

WORCESTER CHAPTER

The Worcester Chapter's first program of the 1957-58 season was a dinner meeting held at the "Hickory House" in September. Mr. Floyd J. Titler, First Vice President of AIIE and Assistant Chief Industrial Engineer of Republic Steel Corporation, presented the charter and spoke on the organizational structure of AIIE.

The October program was a dinner meeting held at the Worcester Polytechnic Institute. Guest speaker was Gordon B. Carson, President of AIIE and Dean of Ohio State University's College of Engineering. Under the title, "New Concepts in Industrial Engineering," Dean Carson pointed out that the Industrial Engineer must prepare himself to make greater use of the applied sciences as tools of analysis, for example, a greater knowledge of 1. statistical mathematics to plan and predict automated production processes and 2. applied psychology to better understand and meet the needs of men, has great urgency as industry becomes more complex and impersonal.

Worcester Chapter officers for the year are: President, Robert J. Hall, Worcester Polytechnic Institute; Vice President, Paul J. Sullivan, Parker and Harper Co.; Program Vice President, Theodore J. England, Norton Company; Secretary, George G. Young, Jr., American Steel & Wire Division, U. S. Steel Corporation; Treasurer, George N. Small, Bay State Abrasives Co.; Directors, Paul L. Lanz, Norton Company; Robert R. Schwarz, American Steel & Wire Division, U. S. Steel Corporation; Raymond Forte, American Optical Co.; George J. Sonntag, Heald Machine Co.

YOUNGSTOWN CHAPTER

Dean Carson spoke to 132 members and friends at the first meeting of the year for the Youngstown Chapter, held September 18, 1957 at the Victoria Cafe.

Graduates of Ohio State were among those attending.

Dean Carson's subject "New Concepts of Industrial Engineering" was well received; his illustrations were very amusing and original. Everyone had a pleasant, informative evening.

Dean Carson started the Youngstown Chapter off on what should be a banner year. Following the Dean in October, John Pillar of Republic Steel will narrate a movie he calls "Visioneering."

In November William Thomas, of Youngstown Sheet and Tube, will give a talk on Work Simplification.

There is no meeting in December. In January one of our local companies will be visited.

In February we are holding a joint meeting with the Mahoning Valley Technical Societies.

In March I.B.M. will devote the entire program to a demonstration of automation equipment. Mr. Jesse Huston, of Youngstown Sheet and Tube, is arranging with I.B.M. for this program.

In April the Youngstown Chapter holds its annual Coffee Symposium and nomination of officers.

In May the annual meeting with installation of officers closes out the year.

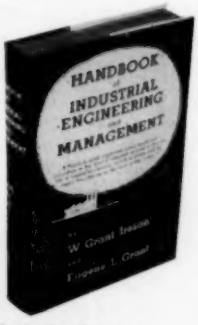
Officers for 1957-58 are President, C. M. Jenkins, Standard Steel Spring, New Castle, Pennsylvania; Vice President, J. C. Talbott, Republic Steel, Youngstown, Ohio; Secretary, F. M. Broberg, Copperweld Steel, Warren, Ohio; Treasurer, R. M. Cashman, Packard Electric, Warren, Ohio.

HANDBOOK OF

Industrial Engineering

AND

Management



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A complete reference book for the industrial engineer, manager, businessman and student on the fields of activity commonly considered to be part of industrial engineering. There are 17 sections in all. Each section is complete in itself and is written by an outstanding authority in the field.

Cross-referenced and fully illustrated, there are sections on: structure of business organization, managerial economics, engineering economy, manpower management, motion and time study, factory systems and procedures, industrial climatology, tool engineering, industrial safety, inspection and quality control, and many others.

1208 pages **6" x 9", double col.** ***$16.00***

Order from:

Journal of Industrial Engineering
225 North Ave., N. W.
Atlanta 13, Georgia

Recent Readables

AUTOMATION

"Computers Control Warehouse System," by Editors, MODERN MATERIALS HANDLING, *page 150, June 1957.*

A completely new concept of automated warehousing based on an over-all "system" approach means faster service, lower costs, and fewer errors in warehousing operations at Mangel's in New York. Highlights of the over-all warehouse system discussed in the article are:

1. Automatic controls for directing order-accumulation operations.
2. A case sealer that adjusts automatically to five different sizes of shipping container.
3. An automatic tally that spots human errors in routing before shipments are made.
4. Controls for sorting outgoing shipments.
5. An automatic set-up that stacks cartons six high on accumulation lines on the shipping dock. This saves dock space.
6. A mechanized system for paper work that eliminates delays from this source.
7. Provision for expansion within the present building and present system.

"Computerized Production Control," Feature, FACTORY MANAGEMENT AND MAINTENANCE, *page 84, July, 1957.*

At the IBM manufacturing plant in Poughkeepsie, New York, 6,000 employees make accounting machines, 700 series computers and miscellaneous civilian and military equipment and components—all requiring 110,000 parts.

Nearly 100 per cent of the major production control and planning is done by a 705 computer. This includes preparation of control information, material breakdown, inventory control, ordering, manufacturing planning and scheduling and follow-up.

"Computers Are Ready to Roll," by Editors, ENGINEERING NEWS-RECORD, *page 21, September 26, 1957.*

Nearly 700 registrants at the latest in a series of highway engineering productivity conferences heard and saw evidence that electronic computers were producing solid results for a surprisingly large number of users.

The unchallenged opinion of several conference speakers was that highway consultants will soon be forced out of business if they don't adopt high speed computation methods.

Demonstrations of M.I.T.'s digital terrain concept of highway location impressed onlookers. The unveiling of the first commercial automatic line plotters (by two equipment makers) was a conference highlight. These machines will draw a straight line between two points, enabling reproduction without human intervention of highway cross sections, profiles and traverses directly from the output of electronic computers, or approximately equipped stereoscopic plotting machines.

GENERAL

"The Use of Psychology in Industry: A Trade Union Point of View," by William Gomberg, MANAGEMENT SCIENCE, *page 348, July, 1957.*

The point of view taken in this paper is that there are elements of identity of interest between management and labor and elements of conflict of interest; that the proper use of psychology in this area calls for the frank recognition of these differences. Psychologists are completely free to use these techniques as long as they forewarn their subjects exactly what objective and whose purpose they are serving.

The author examines the industrial arena in which the psychologist is active. A short excursion is taken into the fields of aptitude testing and the psychological evaluation of Industrial Engineering techniques. However, the major portion is reserved for the human relations area.

"The American Office: Today and Tomorrow," by Thomas Kenney, DUN'S REVIEW AND MODERN INDUSTRY, *page 65, October, 1957.*

This seven part article deals with the latest development in the field. Subjects covered are:

1. Inside the Improved Office—Photostories of offices that have developed new solutions for old problems.
2. Here's How the Future Office Will Look—What specific new equipment do executives want to improve the office? Several designers provide a peek at the office of 1967.
3. The Office Can be a Comedy of Errors—A roundup of actual mistakes companies make in planning new or remodeled offices.
4. Where Office Automation Stands—How deeply have electronic computers penetrated the office? What tasks are they being used for? What have been the results to date?
5. We Needn't Drown in Paper—How can a company stem the rising flood of paperwork? Here are specific tips on cutting down paperwork, improving information.
6. How Tight is the "White Collar Shortage"?—A survey of personnel departments across the nation points up the problems they are facing and their search for solutions.
7. How Fares the White-Collariet?—Is the white collar worker as badly off as some critics claim? Has inflation hurt him more than the blue collar worker in the plant?

"Tools for Alleviating the Engineering Shortage," by G. William King, MACHINE DESIGN, *page 73, July 25, 1957.*

Is the engineering shortage a lack of manpower or is it inefficient use of available manpower? Regardless of the answer to this debate of the day, more true engineering capacity is certainly needed.

An eventual increase in the total number of available engineers is of small help to the company that needs more engineering capacity immediately. Increasing engineering efficiency is the only practical answer for the near future.

The author suggests a critical review of engineering versus non-engineering duties of engineers with a view toward improved efficiency, better morale, and higher professional status.

"Developments in the Analysis Maintenance Problems," by J. D. Quinn, MECHANICAL ENGINEERING, *page 931, October, 1957.*

The days of "fire fighting" in maintenance are rapidly passing, but one cannot be complacent about preventive maintenance

either. Work may be performed conscientiously that better analysis would prove unnecessary.

A logical approach leading to an optimum balance between excessive emergency maintenance and excessive preventive maintenance, includes:

1. A systematic means of procuring background data concerning equipment repairs, their cause and effects.

2. An organized plan for the solution of problems which includes assignment of personnel to the program, and periodic reporting of the program to establish justification and priority for studies.

3. Accumulation of sufficient data to evaluate improvements being considered and, later, to establish the effect of the improvements and the benefits of the over-all program.

Through this approach, the justification of engineering time for problem solution can be established. Problem solution, or corrective maintenance, can be considered as a business proposition that shows an excellent return on the expenditure, and results in a significant contribution to the production of a quality product at a lower cost.

"Charting Techniques That Serve Your Purpose," by W. D. Clarke, OFFICE EXECUTIVE, *page 23, September, 1957.*

In procedure studies, charts are road maps. And procedure analysts, like road surveyors, cut out needless miles of travel and hours of work by charting new systems to eliminate curves and bottlenecks. Like surveyors, the analysts have to overcome the objections of persons whose way of life may be changed by a new procedure. Similarly, analysts must have the support of their superiors in making the changes.

However, to obtain this support the analysts have to sell the advantages of the recommended changes to the persons involved. For this purpose, charts of the old and new procedures are excellent devices.

The flow process chart can be used to record the details of a job in the order in which they occur. This chart is a graphic presentation of the sequence of events in any process or procedure. In fact, the chart is the yardstick with which a job can be measured. Once measured, it can be compared; then improvements can be made.

"Improving Organization and Performance of an Industrial Engineering Department," by M. E. Mundel, ADVANCED MANAGEMENT, *page 8, July, 1957.*

The author discusses how the performance of an Industrial Engineering Group may be improved by:

1. Recognition of the staff nature of the function and the nature of true staff work.

2. Selection of the proper scope of activity.

3. Adequate manning and continual improvement of staff.

4. Attachment in the organization to the decision-making level served.

5. The design and use of a suitable periodic reporting or "self-auditing" system.

"Punched Card Systems Don't Have to be Expensive," by R. Neumaier, THE OFFICE, *page 106, May, 1957.*

Generally speaking, most punched card installations are out of the reach of the small office. To prove that there are exceptions, however, the author cites a few examples where the cost is definitely held within the limitations of a good budget for clerical expense.

The case histories include:

1. A community nursing association.
2. Inventory control in a contracting firm.
3. Inventory forecasting in a metal manufacting plant.
4. Inventory control in a furniture chain.
5. Sales analysis in a service organization.

"Two Applications of a Digital Computer to Machine Design Problems," by J. T. Lester, Jr., TRANSACTIONS OF THE ASME, *page 1476, October, 1957.*

The two examples presented in this paper illustrate the two chief ways in which a high-speed computer can aid the design engineer. The first example, preparation of design tables for the cycloidal-crank drive, was representative of the many straightforward, yet tedious, calculations required in machine design. For such problems the computer can effect important savings in time and money. The second example, the differential equations of motion for a hammer mill, represents the "impossible" type of problem which the computer has rendered solvable on a practical basis.

It is for this latter type of problem that computers have opened up new horizons for the designer. With computers complex mechanisms can be analyzed for high-speed and high-performance operation while they are still on the drawing board and thus eliminate many of the costly trial-and-error methods used in the past.

MATERIALS HANDLING

"Vacuum Lifting: New Overhead Handling Method," by Editors, MODERN MATERIALS HANDLING, *page 128, November, 1957.*

A new dimension has been added to overhead handling by vacuum lifting. It now is possible to lift sheet, plate, concrete slabs, glass sheet and other materials without the use of hooks, slings or loops. What is more, vacuum lifting provides a new tool for automating in-process handling. This roundup, prepared by the editors will help decide whether or not vacuum lifting fits into your operating scheme. It describes vacuum lifting systems as well as the equipment that goes into them.

"Controlling Handling Costs," by R. J. Williams, MODERN MATERIALS HANDLING, *page 137, June, 1957.*

Management at one company felt that the recommended equipment and layout of the new handling setup were but tools. They could not, of themselves, produce savings unless properly used. To insure their proper use, a major portion of their materials handling manual—describing how the operations are to be performed—was devoted to costs control.

Cost information for each operation is accumulated by the handling crew performing the job. Final result is a detailed monthly report on each specific handling operation. Work orders are sent to the Chicago office where dollar cost and man hours employed are computed on the face of the form. At the end of the month the total dollar cost of the operation and the total number of units and weight handled is determined. This information is then posted to the Newark ledger and the average per unit determined.

With this type of information available to the mill manager, it is easy for him to quickly determine if an operation is being performed the way it should. Any trend is easily recognizable.

OPERATIONS RESEARCH

"Applications of Linear Programming in the Oil Industry," by W. W. Garvin, H. W. Crandall, J. B. John, and R. A. Spellman, MANAGEMENT SCIENCE, *page 407, July, 1957.*

This paper is the result of a survey made during the summer of 1956. It is a progress report on applications of linear programming by a number of oil companies. Examples are presented of applications to a variety of problems arising in the areas of Drilling and Production, Manufacturing, and Marketing and Distribution. The examples were selected to illustrate both the power and the limitations of present linear programming methods when applied to actual problems.

STATISTICAL QUALITY CONTROL

"An Application of Statistical Control Techniques to a Post Office Processing Operation," by J. P. Ozenham, INDUSTRIAL QUALITY CONTROL, *page 5, September, 1957.*

In the spring of 1953 the Post Office Department initiated a study of the various methods in use in private industry to effect improvement in operational methods. This study embraced consideration of control charts and related techniques as employed in "Quality Control" activities. Applications of statistical inference and control chart methods were made to an individual postal processing operation at each of four selected post offices. The applied control chart techniques and the data obtained at one office pursuant to these studies are the subject of this article.

"Control Charts in Multi-Stage Batch Processes," by R. S. Bingham, Jr., INDUSTRIAL QUALITY CONTROL, *page 21, June, 1957.*

Application of statistical control in the chemical process industry has been hampered partly by lack of understanding of how statistical concepts may be applied to advantage beyond that commonly gained with standard chemical analyses and physical tests.

This article reviews underlying concepts of statistical control as applied to batch chemical processes. A study of spent acid reduction in a counter-current 3-stage nitration process illustrates some of the principles.

"Significance Tests and The Dollar Sign," by F. C. Leone, R. S Nottingham, J. Zucker, INDUSTRIAL QUALITY CONTROL, *page 5, June, 1957.*

Standard tests of statistical significance of differences between samples of known and unknown populations are among the most important tools of the quality control engineer. The conventional and rigorous application of these tests is generally understood. However, these tests as usually applied have two major deficiencies: 1. Statistical significance does not always represent monetary significance and 2. In the instance where statistical significance is not established, nothing is accomplished except our knowledge of the failure of the experiment to reveal a significant difference, and we are often left in an inconclusive and unsatisfying position. By the application of known statistical methods to the data involved in the performance of these tests the authors state that it is possible to go a long way toward establishing rigorous limits for conclusions even in the case of nonsignificance.

BOOKS

A Management Guide to Electronic Computers, *by William D. Bell, 402 pp., published by McGraw-Hill Book Co., Inc., 330 West 42nd St., New York 36, New York, $6.50.*

This book explains in nontechnical language what computers are for, what the equipment is and how it works, what results and problems the machines bring, how to find if such a system is needed, how to justify it, how to choose and install a system, and actual experiences of any other companies with electronic data-processing.

Integrated Data Processing for Every Office, *by H. John Ross, Office Research Institute, Box 8275, University Station, Coral Gables, Florida, 80 pp., $2.*

Presents the thesis that a simple change in the concept of paper work on the part of top management would bring any company an immediate and inevitable reduction in clerical costs. This reduction conceivably could be effected without expensive, automatic data processing equipment.

Materials Handling Equipment, *by D. Oliphant Haynes, Chilton Co., New York, 636 pp., $17.50.*

Here is a graphic presentation of materials handling equipment in a book designed to show specifically how each machine functions and what it actually does. Over 2,200 line drawings are used to illustrate the basic internal parts, not just the exterior of materials handling machines.

The author elaborates in detail on transporting, elevating, conveying and transferring equipment and on the handling procedures used with each type. This, coupled with a complete rundown on self-loading, bulk-handling and accessory equipment gives to the reader a neat package of handling hints.

Professional Engineers' Income and Salary Survey, 1956, *40 pp., available from the National Society of Professional Engineers, 2029 K St., N.W., Washington 6, D.C., 50 cents per copy to NSPE members, $1.00 per copy to nonmembers.*

This report represents findings of the third national salary survey conducted by the National Society of Professional Engineers. The present survey compares data with previous surveys and shows salary trends since 1952.

Information was compiled on the basis of questionnaires returned by members of NSPE. Numerous charts and tables show earnings according to geographic distribution of the respondents, year of entry into the profession, branch of engineering, and kind of work. Also included are income differences between engineers in industry and other fields of employment.

Past Examinations for Professional Engineer, *1957 Edition, 81 pp., paperbound, published by and available from John D. Comstance, P. E., 625 Hudson Ter., Cliffside Park, New Jersey, $2.00 per copy.*

This is a compilation of past examinations for Professional Engineer given by the New York State Board of Examiners, and covers years 1946-1957.

Three major parts include structural planning and design, basic engineering sciences, and engineering economics and practice.

Designing for Production, *by Edward N. Baldwin and Benjamin W. Niebel, 645 pp., published by Richard D. Irwin Inc., Homewood, Illinois, $8.50 postpaid.*

Engineering materials and processes are described in this book from the designer's viewpoint. Information is presented so that designers can select materials best suited for design with respect to processing, service life, first cost, maintenance requirements and function.

Principal manufacturing processes including metal forming and shaping, cutting, jointing, finishing, inspection, assembly, and packaging are described and analyzed.

A History of Mechanics, *by Rene Dugas, Ecole Polytechnique, Paris, translated by J. R. Maddox, 671 pp., published by The Central Book Co., Inc., $15.*

This book presents, in a chronological order, the evolution of principles of general mechanics from the time of Aristotle to the present day. It is divided into five parts. The first deals with the origins of mechanics and includes Hellenic science, Alexandrian sources and Arabic manuscripts, and various schools of thought up to and including the 16th century. Part two, formation of classical mechanics, presents theories and principles of Galileo, Torricelli, Roberval, Descartes, Huygens, Newton and others. Part three pertains to 18th century mechanics up to Lagrange, and part four presents characteristic features of evolution of mechanics after Lagrange. Part five contains principles of modern physical theories of mechanisms and is divided into sections on presentation of the development of quantum mechanics by early researchers. Concluding sections of part five analyze and interpret each of these theories in the light of more recent developments.

opportunities

WHAT IT IS

The I.E. Opportunities service is a functional committee activity of the AIIE. Information concerning employment opportunities is collected on a nationwide basis and is provided without charge to members upon request. The committee is composed of members from the Columbus, Ohio chapter.

SERVICES PROVIDED

Current job openings are published in condensed form in each issue.

In addition, a monthly I. E. Opportunities Bulletin is supplied to over seventy chapters located throughout the United States.

Each job opening is assigned a "**P**" number for identification. The name and address of the person to contact for additional information concerning a specific job opening is sent to members on request.

EMPLOYERS

Employers having openings for qualified Industrial Engineers are invited to list them. Government agencies and educational institutions as well as business and industry are urged to take advantage of this free service.

Industrial Engineers are often employed in such fields as Work Measurement, Production Control, Plant Engineering, Operations Research, Industrial Relations, Sales, and Management. For a more complete coverage, see the following list of job classifications. Please do not hesitate to forward information on jobs in any area where it is recognized that an Industrial Engineering background will be of value.

To list your job openings with the Opportunities Service, send all or part of the following information with the name of your contact man to the address below: Type of industry, location, job classifications, minimum educational and experience qualifications, and salary range.

This information will be publicized to the members in condensed form in the JOURNAL and in monthly chapter Bulletins. Company names are not shown.

MEMBERS SEEKING JOB OPPORTUNITIES

The following list shows job openings available just prior to press time. If you would like more information about one or more of the positions listed, mail the "**P**" numbers with your name and address to the Opportunities Service at the address below. The Service will advise you by return mail of the name and address of the person to contact for further information, or advise you if the position is no longer available. Your name will not be forwarded to the company with the job opening.

For more current listings, contact your local Chapter Secretary or Opportunities Chairman for the latest monthly Bulletin.

ADDRESS OF THE SERVICE

I. E. OPPORTUNITIES SERVICE
American Institute of Industrial Engineers
145 North High Street
Columbus 15, Ohio

INDUSTRIAL ENGINEERING OPPORTUNITIES

Position number	Industry, location		Travel?	Job classification number (See key)	Salary	Qualifications required			
					Range in $1,000	Adv. degree req.?	Years of exper.	Age range	
								from	to
68	Education	Pa		10, 20, 31, 42					
69	Steel Products	W Va		26					
70	Education	NY		50		Ph.D.			
71	Mgmt. Consult.	Ill	×	13, 21, 38, 42, 47, 70					
72	Airline	Ill		70, 42, 43					
73	Consultant	Pa	×	13, 21					
75	Army Ord.	Ill		13, 31, 37, 70	7–9		2–4		
76	Trans.	Panama		48	9.5–				
77	Post Office	Minn		92—38, 40, 13	7.7–8.5		4		
78	Mining & Smelt.	NM		91—(10, 20, 30, 40)	8–11.1				45
79	Mining & Smelt.	NM		10, 20, 30, 40			6–9.4		45
81	Household Prod.	O		70, 38, 50, 11	+R		0–5		
83	Textile Co.	Ga		37, 42, 38	4.8–7.5				30
84	Education	NY		50 or 60	5.0–9.0				
86	Public Utility	DC		13, 18, 37, 50, 52					
87	Linoleum Mfg.	Pa		42, 26, 11, 53, 81			0–3		
89	Foundry	Cal							
90	Education	Fla							
91	Syn. Fibers & Films	Pa, Va, W Va		10, 20, 42, 43, 72			0–5		
93	Foundry	Ill		11, 19, 30					
95	Airframe Mfg.	Midwest		92—13, 19, 11, 40			5	30	40
96	Navy	Md		40	7.5–8.6		3.5		
97	Railroad	Md		54, 50, 11	8.4				
98	Timber Mill	N West	×	91—30, 40				35	
99	Process	NJ		92—30	12–15			35	45
100	Manufacturing	Ill		95	15+B			37	42
101	Air Force	Ala		13, 11, 16	5.4–7.5		2		
104	Rug Mill	Pa		11, 13, 21, 42	6.0–7.0				
105	Electrical	Mass		92 or 37 or 11	6.5–11.0				

Position number	Industry, location		Travel?	Job classification number (See key)	Salary	Qualifications required			
					Range in $1,000	Adv. degree req.?	Years of exper.	Age range	
								from	to
106	Sugar	ND, La, SC, Conn		13, 43			0		
107	Aircraft	Wash		13, 30					
108	Mgmt. Consult	Overseas	×	10, 20, 60			10	35	40
109	Paper	NH			6.0–6.5		108		
110	Aircraft	Tex		11, 13, 19, 70, 33, 38	5.4–6.6				
111	Chemical	Ala			7.5				
112	Aircraft	Tex		11, 40, 70					
113	Chemicals	NJ		52					
114	Paper	Mo		11					
115	Foundry, Educ.	Pa							
116	Instruments	NY		71	8.0				
117	Electrical	Mass		50					
118	Mfg.	Pa		11, or 16 or 37	7.0–8.0		4–8		
119	Paper	Ga		10, 20, 30, 42 43, 53	6.0–7.2				
120	Air Force	Ga		I.E.	7.0				
121	Navy	Cal		11, 13, 42, 43, 53	6.1–7.6		2–6		
122	Navy	Va		I.E.					
123	Printing	NY		49, 53			2–4		
124	Mineral Prod.	NJ		11, 53			0–3		
125	Chemical	Mich		13, 21, 43, 33			1–5		
126	Boilers-Fdry.	O		10, 20, 30, 42, 43, 44	7.2			25	35
127	Theatrical	NY		42, 43, 45, 49	10–12		5–		
128	Cartons	Conn			4.8–5.9		0–3		35
129	Education	ND		Asst. Prof. I.E.					
130	Heat Exchangers	NY		92—37	8.0–10.0				
131	Radiators	Md		44, 48	6.5–7.6				
132	Oil Seals	O		10, 20, 42, 45, 53	6.0		2–	25	30
133	Flour Mills	Minn	×	11, 13, 42, 43, 94	7.2–8.4				40
134	Education*	W Va		52, 53					
135	Electrochemical	Midwest		92—80	15–20				40
136	Hospital	Pa		11, 63	6.0–8.0		2–		
137	Copper†	Chile, S A	×	31, 39, 43, 64	9.0				
138	Wax	Wisc		11, 19, 42, 47, 81	5.9–6.9		2–4		
139	Pharmaceuticals	Ind		11, 13, 21, 42, 44					
140	Magazine-Editorial	Mass		43	5.0–7.0				
141	Steam Electric Power	Toronto		93—80	11.0		13 (P.E.)	35	45
142	Consumer Products	Midwest		95	15+Bonus			25	40
143	Construction	NY		92—36, 38	11.0–14.0			32	45
144	Aircraft	Texas		15, 19, 39, 42, 44, 53	4.8–6.4		2–5	22	45
145	Lithography	Vt		31			2		
147	Paper & Bag Mfr.	Ala		10, 20, 35, 43, 54, 70					
148	Steel Mill	Italy	×	30, 42	12			30	40
149	Agricultural Marketing	Wash, DC	×	10, 42, 43, 53	6.1		1–		
150	Aircraft Engines	NJ		11, 18, 38, 47, 70, 90			5–		
151	Medical Supplies	NJ		11, 21, 30, 42, 43, 50					
152	Electronics	Ky		10, 32, 39, 40	4.8–7.2		2–5		
153	Woodworking Tools	Texas							
154	Metal Furniture	Va		30, 36, 44			5–10	30	45
155	Electronics	Texas		11, 13			3–		
156	Plastics	Texas							
157	Aircraft	Colo		Mgmt. Engrs.			5–		
158	Pyrotechnics	O		13, 21			2–		
159	Acct'g	Ill, Calif, Ga	×	26, 32, 36, 42, 43, 70	–14.0		5–	30	40
161	Printing	Midw		Asst. to Prod. V.P.	10.0–13.0			28	38
162	Ordnance	Hawaii		11	GS-11+20%				
163	Govnmt Inspection	O		37	64.–7.5		7–		
164	Precision Mach Shop	Texas		11, 13, 19, 32, 42	6.0–7.2		3–5		35
165	Electro-Mechanical	Calif		11, 13, 19, 42, 53	5.8–		0–3		
166	Paper Mill	Pa		91–16			4–5		
167	Weapons Systems	Wash DC		50		Ph.D.			
168	Education	Mich		92	6.0–7.5	Ph.D./M.S.	1–5	25	55
169	Rubber & Plastics	Ind		10, 22, 23, 36, 42, 43	7.0		7–10		45
170	Rubber & Plastics	Ind		91–10, 20, 34, 36, 42					50
				43, 70	8.0–9.0		7–15		50
171	Roller Bearings	O		10 Thru 28	5.0–6.0		0–2		
172	Power Shovels	Pa, Wisc, Ind		11, 13, 21, 32, 39					
173	Education	Pa		19, 53, 51	6.0–10.0	Ph.D./M.S.	1–5		
174	Highway Dept	Mich		11, 32, 38, 45, 70	5.3–7.1		2		
175	Printing	O		11, 13, 21, 36					

* Begins Sept.
† 3 yr contract.

Key to Job Classifications

No.	Job Classification
10	Motion and Time Study
11	Methods Improvement
12	Suggestion Systems
13	Work Measurement and Perf Stds.
14	Stop Watch Time Study
15	Std. Time Data Dev. & Applic.
16	Predeter. Elemental Time Stds.
18	Work Sampling
19	Estimating and Costing
20	Wage Payment
21	Incentive Plans
22	For Production Workers
23	For Non-Prod. Workers
24	For Supervisory Personnel
6	Job Evaluation
8	Wage Administration
30	Production Engineering
31	Production Control
32	Process Planning and Routing
33	Scheduling and Loading
34	Flow Process Charting
35	Inventory Control
36	Cost Anal. & Reduction
37	Statistical Quality Control
38	Budgetary Control, Standard Costs
39	Tool and Gage Design and Control
40	Plant Engineering
41	Plant Location & Expansion
42	Plant Layout
43	Material Handling
44	Machinery & Equipment
45	Specif., Select. & Eval.
46	Replacement
47	Automation
48	Plant Maintenance
49	Cap. Budget. Facil. Plan
50	Operations Research
51	System & Simulation with "Models"
52	Mathematical Analysis
53	Engr. Economy Studies
54	Auto. Data Proc. with Computers
55	Market Research & Forecasting
60	Industrial Relations
61	Personnel Administration
62	Personnel Testing
63	Personnel Training
64	Industrial Psychology
65	Labor Relations
66	Safety Engineering
67	Suggestion Systems
70	Systems and Procedures
71	Admin. & Operating Procedures
72	Organization Charts and Manuals
73	Records Admin. & Form Control
80	Product Design
81	Packaging
90	Management and Supervision
91	Industrial Engineering Supr.
92	Chief I. E. or Equiv.
93	Plant Engineer
94	Production Supervisor
95	Plant Mgr., Fact. Mgr., Works Mgr.
96	General Manager

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Industrial Engineering is concerned with the design, improvement, and installation of integrated systems of men, materials and equipment; drawing upon specialized knowledge and skill in the mathematical, physical, and social sciences together with the principles and methods of engineering analysis and design, to specify, predict, and evaluate the results to be obtained from such systems.